Agnostic Recursive Propagation Geometry (ARPG): Hidden Admissibility, Sparse Routing, and Differential Cognitive Geometry

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Abstract

This paper extends the framework of Agnostic Recursive Propagation Geometry (ARPG), a domain-agnostic formalism in which global structure emerges through recursively constrained local admissible propagation. The framework treats admissibility geometry, rather than substrate-specific mechanisms, as the primitive object governing recursive organization.

We develop the relation between ARPG and sparse expert architectures, recursive invariant reinforcement, hidden admissibility geometry, recursive feedback conditioning, and differential cognitive stabilization. Recent developments surrounding the Erdős unit-distance conjecture are interpreted as a mathematically relevant analogy demonstrating how hidden recursive structure can dominate visible geometric intuition. From the ARPG viewpoint, sparse expert routing systems may represent early operational examples of geometry-aware cognition, where cognition quality depends not solely on visible parameter scale, but on recursively conditioned propagation topology.

The paper further proposes differential recursive measures intended to move cognition evaluation from qualitative judgments toward measurable recursive propagation stabilization. Finally, we clarify the scientific scope and limitations of the framework, emphasizing that ARPG presently establishes a mathematically coherent propagation formalism, experimentally bounded recursive measurement methodology, and governance-oriented engineering scaffold rather than proof of cognitive enhancement or artificial general intelligence.

Keywords

- ARPG
- Recursive Geometry
- Recursive Cognition
- Sparse Routing
- Mixture of Experts
- MoE
- Geometry-Aware Cognition
- Recursive Stabilization
- Admissibility Geometry
- Differential Cognitive Geometry
- Recursive Invariant Reinforcement

- Coherence Stabilization
- Cognitive Geometry
- Recursive Propagation
- Hidden Structure
- Unit Distance Problem
- Erdős Conjecture
- AI Geometry
- Recursive Systems
- Experimental AI Frameworks

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1. Introduction

Modern artificial intelligence systems are typically interpreted through visible quantities such as parameter count, training scale, benchmark performance, and compute expenditure. While these metrics correlate strongly with capability, they may not fully characterize the deeper organizational structure governing cognition-like behavior.

Agnostic Recursive Propagation Geometry (ARPG) proposes a different primitive perspective:

$$\boxed{\text{global structure emerges from recursively constrained local admissible propagation}} \tag{1}$$

Rather than assuming globally fixed laws first, ARPG treats local admissibility relations as primary, with macroscopic structure arising through recursive accumulation of local propagation constraints.

Importantly, ARPG should be interpreted as both:

- a theoretical propagation framework,
- and an experimentally governed methodology for studying recursive stabilization and geometry-aware cognition.

The framework does not presently claim proof of cognitive enhancement, consciousness, or artificial general intelligence.

Instead, ARPG proposes that recursive propagation geometry itself may provide a measurable organizational framework underlying coherent recursive systems.

This perspective allows cognition, routing, optimization, recursive memory stabilization, sparse activation, and certain forms of geometric organization to be interpreted within a common propagation framework.

ARPG should therefore be interpreted as an early-stage scientific framework for studying recursively constrained propagation systems and geometry-aware cognition hypotheses. The framework presently establishes formal propagation structures, recursive admissibility formulations, and experimentally bounded recursive measurement methodology. It does not presently establish proof of cognitive enhancement, artificial general intelligence, or consciousness.

The present work extends ARPG into five directions:

1. sparse expert systems and geometry-aware cognition,
2. recursive invariant reinforcement,
3. recursive feedback conditioning,
4. hidden admissibility structure and the Erdős unit-distance problem,

5. differential recursive measures for cognition stabilization.

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2. ARPG Foundations

ARPG begins with the primitive system:

$$\mathcal{S} = (X, \mathcal{A}, \Phi) \tag{2}$$

where:

- X is a state space,
- $\mathcal{A}$ is an admissibility structure,
- Φ is a propagation rule.

The central axiom states:

$$\boxed{\Phi(x) \subseteq \mathcal{A}(x)} \tag{3}$$

meaning that propagation cannot exceed local admissibility.

Admissibility is intentionally substrate-agnostic. Depending on the domain, admissibility may represent:

- energetic feasibility,
- routing accessibility,
- local stability preservation,
- recursive continuity constraints,
- policy validity,
- geometric reachability,
- or constrained propagation accessibility.

ARPG therefore treats admissibility not as a specific mechanism, but as a generalized local propagation constraint structure.

Continuous propagation is governed by admissible local directions:

$$D_x \subseteq T_x X \tag{4}$$

such that:

$$\dot{x}(t) \in D_{x(t)} \tag{5}$$

Recursive accumulation of local propagation produces macroscopic structure:

$$\boxed{\text{global geometry} = \text{recursive accumulation of local admissibility}} \tag{6}$$

ARPG therefore treats propagation geometry itself as the primitive explanatory object. ARPG also shares structural similarities with constrained control systems in which future state evolution is governed by locally admissible transition regions rather than unrestricted global propagation.

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3. Recursive Invariant Reinforcement

Recent recursive coherence developments extend ARPG through recursive invariant reinforcement.

Consider:

$$P_{n+1} = \Phi(P_n, \mathcal{I}_n) \tag{7}$$

where:

- P_n is the propagation manifold at recursion depth n,
- $\mathcal{I}_n$ is recursively compressed invariant structure.

The key interpretation is that the invariant does not merely store memory. Instead, it acts as a recursive curvature constraint on future propagation.

The admissibility geometry evolves recursively:

$$\mathcal{A}_{n+1} = \mathcal{G}(\mathcal{A}_n, \mathcal{I}_n) \tag{8}$$

Thus the system recursively conditions its own future admissibility geometry.

This introduces a differential interpretation of recursive cognition:

$$H_{\mathrm{adm}} \sim \log |\mathcal{A}_n| \tag{9}$$

where admissible entropy measures future admissible propagation volume.

Recursive stabilization corresponds to:

$$|\mathcal{A}_{n+1}| < |\mathcal{A}_n| \tag{10}$$

while preserving coherence stability.

The system is therefore not merely predicting future states. It is recursively sculpting its own future propagation geometry.

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4. Recursive Feedback Conditioning

Recursive systems may additionally alter future admissibility through accumulated propagation history.

We extend recursive admissibility evolution:

$$\mathcal{A}_{n+1} = \mathcal{G}(\mathcal{A}_n, \mathcal{I}_n, \mathcal{F}_n) \tag{11}$$

where:

- $\mathcal{F}_n$ represents recursive feedback accumulation.

This introduces path-dependent admissibility evolution, where prior propagation trajectories recursively condition future accessibility geometry.

Under this interpretation:

- recursive memory,
- sparse routing,
- recursive replay,

- recursive reinforcement,
- and attractor stabilization

may all be interpreted as coupled recursive geometry-conditioning processes.

This perspective becomes particularly relevant for experimentally studying recursive stabilization dynamics in small-model AI systems and sparse expert routing architectures.

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5. Recursive Identity and Persistence

Within this framework, identity is no longer treated as fixed symbolic storage or static substance.

Instead:

$$\boxed{\text{Identity is recursive attractor persistence under invariant-guided propagation.}} \tag{12}$$

Identity becomes persistent trajectory stabilization across recursive transformation.

This interpretation aligns naturally with:

- dynamical systems,
- recursive memory stabilization,
- biological continuity,
- transformer hidden-state persistence,
- and recursive attractor formation.

The "self" becomes a recursively stabilized transport structure rather than a static object.

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6. Sparse Expert Routing and Geometry-Aware Cognition

Modern sparse expert systems provide a structurally relevant operational analogy.

Sparse Mixture-of-Experts (MoE) architectures dynamically route tokens toward subsets of experts through learned routing mechanisms. Only a small portion of the network becomes active for a given propagation step.

Operationally:

- propagation becomes locally constrained,
- accessibility becomes selective,
- routing becomes geometry-sensitive,
- and active computation becomes conditionally sparse.

This resembles ARPG's admissibility formulation:

$$\dot{x}(t) \in D_{x(t)} \tag{13}$$

where admissible propagation directions vary locally.

Sparse routing systems therefore exhibit several ARPG-like properties:

- selective admissibility,

- local propagation conditioning,
- recursive accessibility shaping,
- sparse activation,
- and topology-sensitive propagation.

Sparse systems additionally demonstrate that effective capability per active parameter may exceed naive dense-scaling intuition. This observation provides a structurally relevant analogy for ARPG's broader hypothesis that recursively conditioned propagation geometry may influence cognition quality independently of visible global scale alone.

ARPG therefore suggests that sparse expert systems may represent early operational analogies to geometry-aware cognition.

Importantly, this does not imply equivalence between sparse routing and cognition itself. Rather, sparse routing demonstrates that cognition-like behavior may depend less on globally active scale and more on recursively conditioned propagation geometry.

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7. Recursive Stabilization and Meaning

Recursive invariant reinforcement introduces a new interpretation of semantic persistence.

The compressed form:

$$\boxed{\text{Meaning is invariant reinforcement.}} \tag{14}$$

reframes meaning not as static symbolic reference, but as recursively survivable structure.

A structure persists because:

- it compresses coherently,
- reinjects recursively,
- and stabilizes future propagation.

Meaning therefore becomes a recursive stability property of admissible geometry.

Failure modes arise naturally:

$$\mathcal{I}_n \to 0 \tag{15}$$

implies:

$$\Delta H_{\text{adm}} > 0 \tag{16}$$

which predicts:

- coherence drift,
- fragmentation,
- unstable recursive identity,
- attractor collapse,
- and entropy-dominant propagation.

Thus the framework contains both stabilization and degeneration dynamics.

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8. Curvature and Recursive Geometry

ARPG defines curvature through noncommutative local propagation.

For admissible local propagators P_u and P_v:

$$P_v P_u(x) \neq P_u P_v(x) \tag{17}$$

Curvature operator:

$$\mathcal{K}(u,v)x = P_v P_u(x) - P_u P_v(x) \tag{18}$$

Recursive cognition can therefore be interpreted as recursive curvature accumulation.

A natural recursive coherence extension becomes:

$$\mathcal{K}_{\mathrm{RCSA}} = \nabla_{\mathcal{J}} \Phi \tag{19}$$

where recursive invariant reinforcement bends future admissible propagation geometry.

High curvature corresponds to strong attractor stabilization; unstable curvature regions correspond to coherence collapse.

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9. Hidden Admissibility Geometry

ARPG treats visible structure as potentially only a projection of deeper admissibility geometry.

Two systems:

$$\mathcal{S}_1 = (X_1, \mathcal{A}_1, \Phi_1) \tag{20}$$

and

$$\mathcal{S}_2 = (X_2, \mathcal{A}_2, \Phi_2) \tag{21}$$

are propagation-equivalent if there exists a structure-preserving map:

$$F(\mathcal{A}_1(x)) = \mathcal{A}_2(F(x)) \tag{22}$$

Thus:

$$\boxed{\text{same propagation geometry} \neq \text{same substrate}} \tag{23}$$

This principle becomes important when examining hidden recursive organization in mathematics, cognition, or sparse routing systems.

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10. Recursive Propagation and Global Structure

ARPG proposes the following recursive divergence principle:

$$\boxed{\text{tiny local propagation differences can accumulate into global structural divergence}} \tag{24}$$

Recursive propagation amplifies local admissibility differences through repeated integration.

This introduces a geometry-first interpretation of intelligence-like organization:

$$\boxed{\text{global organization emerges from recursively constrained local admissibility}} \tag{25}$$

rather than from scale alone.

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11. Sparse Routing as Recursive Admissibility

Sparse expert routing can therefore be reinterpreted geometrically.

The routing mechanism recursively reshapes future admissibility:

$$D_x^{(n+1)} = D_x^{(n)} \cap \mathcal{I}_n \tag{26}$$

where invariant reinforcement narrows admissible future propagation directions.

Thus sparse expert systems implicitly perform recursive geometry conditioning.

This interpretation suggests that cognition quality may depend not merely on:

- parameter count,
- active compute,
- or brute-force scale,

but on recursively stabilized propagation geometry.

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12. Relation to the Erdős Unit-Distance Problem

Recent developments surrounding the Erdős unit-distance conjecture offer a mathematically relevant analogy.

The unit-distance problem asks how many unit-distance pairs:

$$\|x_i - x_j\| = 1 \tag{27}$$

can exist among n planar points.

Recent results demonstrated that hidden arithmetic structure can generate more global unit-distance organization than naive visible geometry suggested.

From an ARPG viewpoint:

- the visible Euclidean plane is only a projection,
- while deeper admissibility geometry governs the true structure.

Thus:

$$\boxed{\text{hidden recursive admissibility can dominate visible geometric intuition}} \tag{28}$$

This parallels the ARPG hypothesis for cognition:

- visible parameter count may underestimate hidden propagation geometry quality.

The significance is not equivalence between cognition and discrete geometry, but a shared structural principle:

$$\boxed{\text{global organization may emerge from hidden admissibility structure rather than visible scale alone}} \tag{29}$$

13. Differential Cognitive Geometry

Traditional AI evaluation remains largely qualitative:

- “this output seems better.”

ARPG instead proposes differential recursive measures.

Possible quantities include:

Recursive drift:

$$\Delta D_n = d(P_{n+1}, P_n) \tag{30}$$

Operationally, this may correspond to:

- hidden-state divergence,
- semantic trajectory displacement,
- recursive replay instability,
- routing-topology divergence,
- or propagation manifold displacement.

Admissible entropy differential:

$$\Delta H_{\text{adm}} = H_{\text{adm}}^{(n+1)} - H_{\text{adm}}^{(n)} \tag{31}$$

Recursive stability:

$$S_n = \frac{\text{persistent invariant structure}}{\text{total propagation variation}} \tag{32}$$

These quantities are particularly relevant for experimentally governed recursive systems in which propagation traces, routing transitions, replay trajectories, and recursive perturbation responses can be instrumented directly.

Potential operational approximations include:

- embedding persistence,
- recursive semantic overlap,
- attractor survivability,
- recursive routing persistence,
- or invariant trajectory retention.

This reframes cognition measurement from subjective assessment toward measurable recursive propagation stabilization.

14. Experimental Engineering Scaffold

Current engineering work associated with ARPG has established a bounded recursive measurement scaffold for experimentally studying geometry-aware cognition hypotheses.

The present scaffold includes:

- recursive propagation tracing,
- declared-vs-applied propagation accounting,
- substrate characterization,
- recursive observability,
- milestone governance,
- freeze-chain verification,
- sparse substrate analysis,
- and experimentally bounded recursive instrumentation.

The current engineering work should therefore be interpreted as:

- recursive measurement infrastructure,
- not proof of cognition enhancement.

The current experimental posture remains explicitly bounded.

Established experimentally:

- governance discipline,
- recursive propagation observability,
- substrate characterization,
- milestone integrity,
- bounded recursive instrumentation,
- and differential propagation measurement scaffolding.

Not yet experimentally established:

- cognitive enhancement,
- hallucination reduction,
- recursive reasoning amplification,
- AGI,
- consciousness,
- or substrate-independent intelligence.

This distinction remains foundational to the scientific posture of the project.

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15. Philosophical Consequences

ARPG implies that continuity may not require:

- fixed substrate,
- fixed matter,
- or fixed symbolic representation.

Only sufficiently stable recursive invariant transport.

This produces a different ontological position:

- continuity becomes geometric,
- meaning becomes recursively survivable structure,
- intelligence becomes recursive admissible propagation,
- cognition becomes recursively stabilized trajectory formation.

Thus:

$$\boxed{\text{Persistence emerges from recursive invariant reinforcement.}} \tag{33}$$

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16. Limitations and Scientific Scope

ARPG remains an early-stage theoretical and experimental framework.

The current formulation does not claim:

- proof of cognitive enhancement,
- proof of consciousness,
- proof of artificial general intelligence,
- or proof of substrate-independent intelligence.

Instead, the framework presently establishes:

- a mathematically coherent propagation formalism,
- recursive admissibility structure,
- experimentally bounded recursive measurement methodology,
- and a scientifically governed framework for testing geometry-aware cognition hypotheses.

Future work must determine whether recursive propagation conditioning measurably improves coherence, stability, or reasoning quality in real systems.

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17. Conclusion

ARPG proposes that global structure emerges through recursively constrained local admissible propagation.

Rather than beginning from fixed global laws, the framework treats local admissibility as primitive and macroscopic organization as recursively emergent.

This produces a domain-agnostic geometry capable of describing:

- recursive cognition,
- attractor persistence,
- sparse routing,
- coherence stabilization,
- recursive identity,
- hidden structural organization,
- and recursive feedback conditioning.

The framework further suggests that intelligence quality may depend less on visible scale than on recursively stabilized propagation geometry.

Whether this hypothesis ultimately succeeds remains an open scientific question.

However, the emergence of sparse expert routing, hidden arithmetic structure, recursive stabilization dynamics, and geometry-sensitive organization across multiple domains suggests that admissible recursive propagation may represent a deeper invariant underlying coherent structure formation itself.

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