Fundamentals of Power System Optimization

Frank A. Felder

frankafelder@independentelectricityconsultants.com

https://independentelectricityconsultants.com

UPCOMING TRAINING

Half-day, Morning Sessions

- Dec. 2: Introduction to the Electricity Sector
- Dec. 3: Power System Fundamentals
- Dec. 9: Electricity Markets
- Dec. 10: Regulatory and Business Context

- Dec. 2: Trump Electricity Policy Update, 2 pm
- Dec. 9: Quiz Bowl, 4 pm

Gatineau & Virtual

Training schedule and slides available at
https://www.independentelectricityconsultants.com/training-presentations

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TABLE OF CONTENTS

What is optimization:

- Objective function
- Constraint equations
- Variables
- Parameters

Case Study #1: Economic dispatch problem

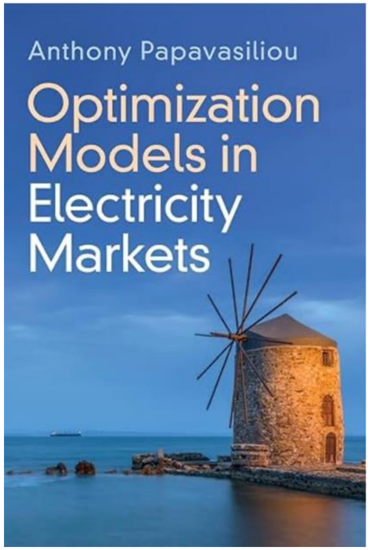
Cast Study #2: 100% carbon-free data center

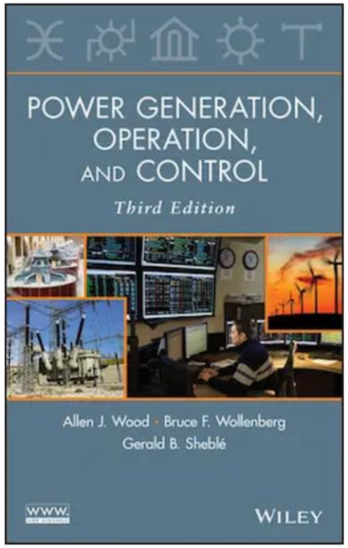
Examples of optimization problems in the electricity sector:

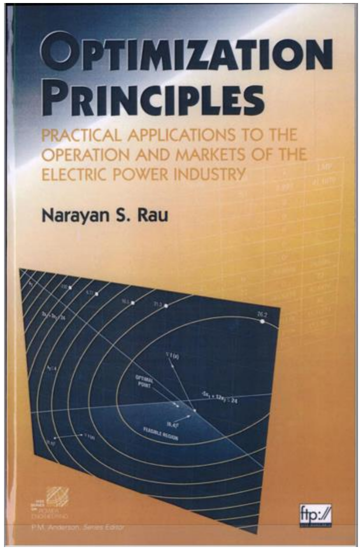
- Unit commitment/day-ahead market
- Generation planned outage scheduling
- Auctioning of financial transmission rights
- Energy storage
- Hydroelectric scheduling and operations
- Generation and transmission expansion planning
- Microgrids

Resources for future professional development:

- Software tools
- Tutorials
- References and resources







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MAJOR THEMES

Optimization is prevalent in the electricity sector and in all economic sectors

Optimization algorithms set prices in multiple electricity markets

Large-scale optimization problems can be solved on personal computers

Framing situations as optimization problems, even if not solved quantitatively, is extremely useful

KEY CONCEPTS

Maximization

Minimization

Categories of optimization problems

Resource constraints

Shadow prices

Sensitivity analysis

Algorithms

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SEMINAR AGENDA

9:30-9:40 Introduction, Logistics, Course Overview

9:40-10:10 Introduction to Optimization

10:10-11:00 Economic Dispatch Case Study

11:00-11:15 Break

11:15-12:00 Data Center Case Study

12:00-12:25 Additional Electric Sector Optimization Applications

12:25 to 12:30 Q&A

Send an email to frankafelder@independentelectricityconsultants.com to request a letter of attendance for professional education requirements

Ask questions or comment at anytime during the session and feel free to contact me at anytime if you would like more information or discussion.

Connect with me on LinkedIn: https://www.linkedin.com/in/frank-felder-8766976/

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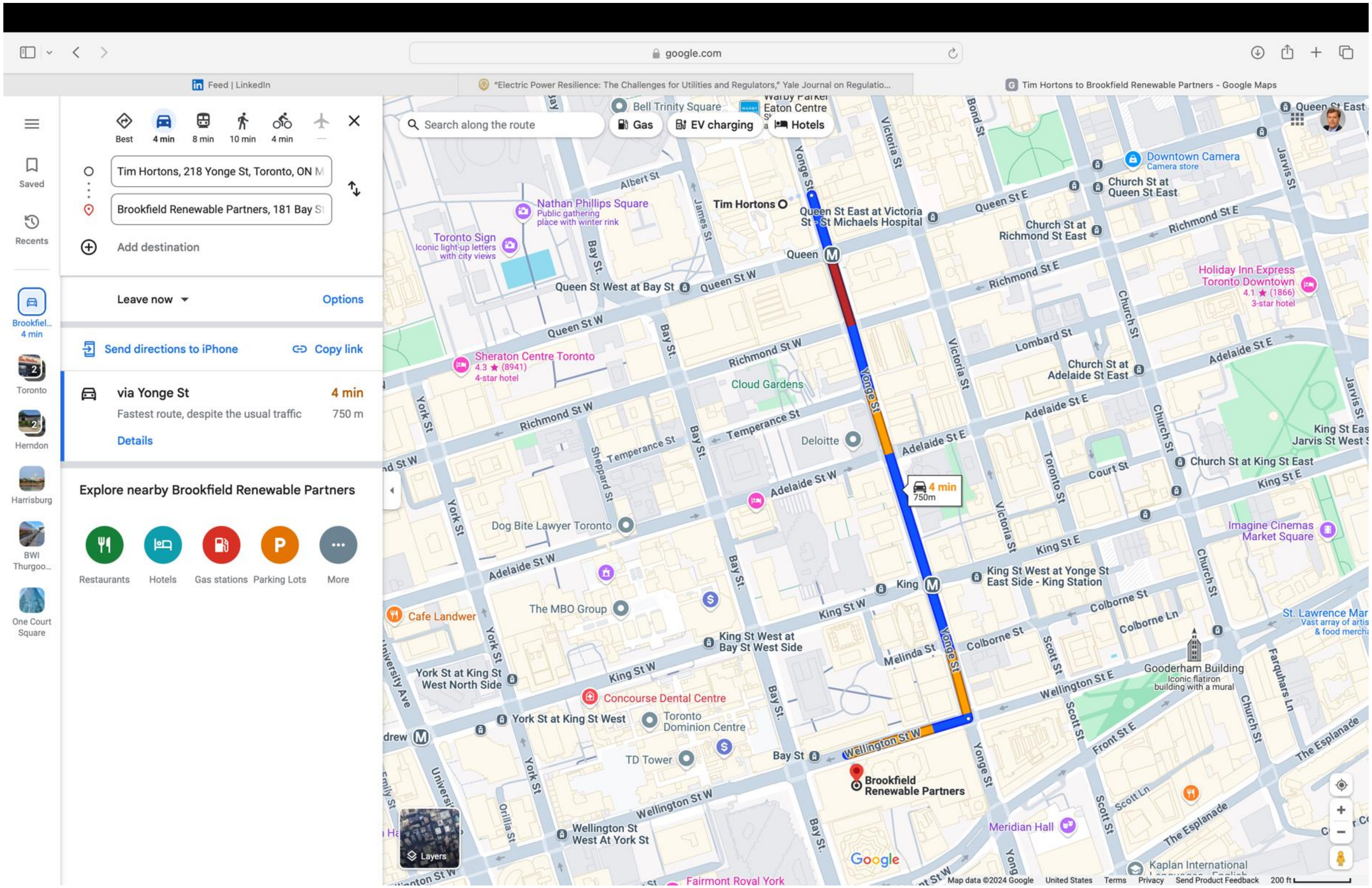
OPTIMIZATION IN EVERYDAY LIFE



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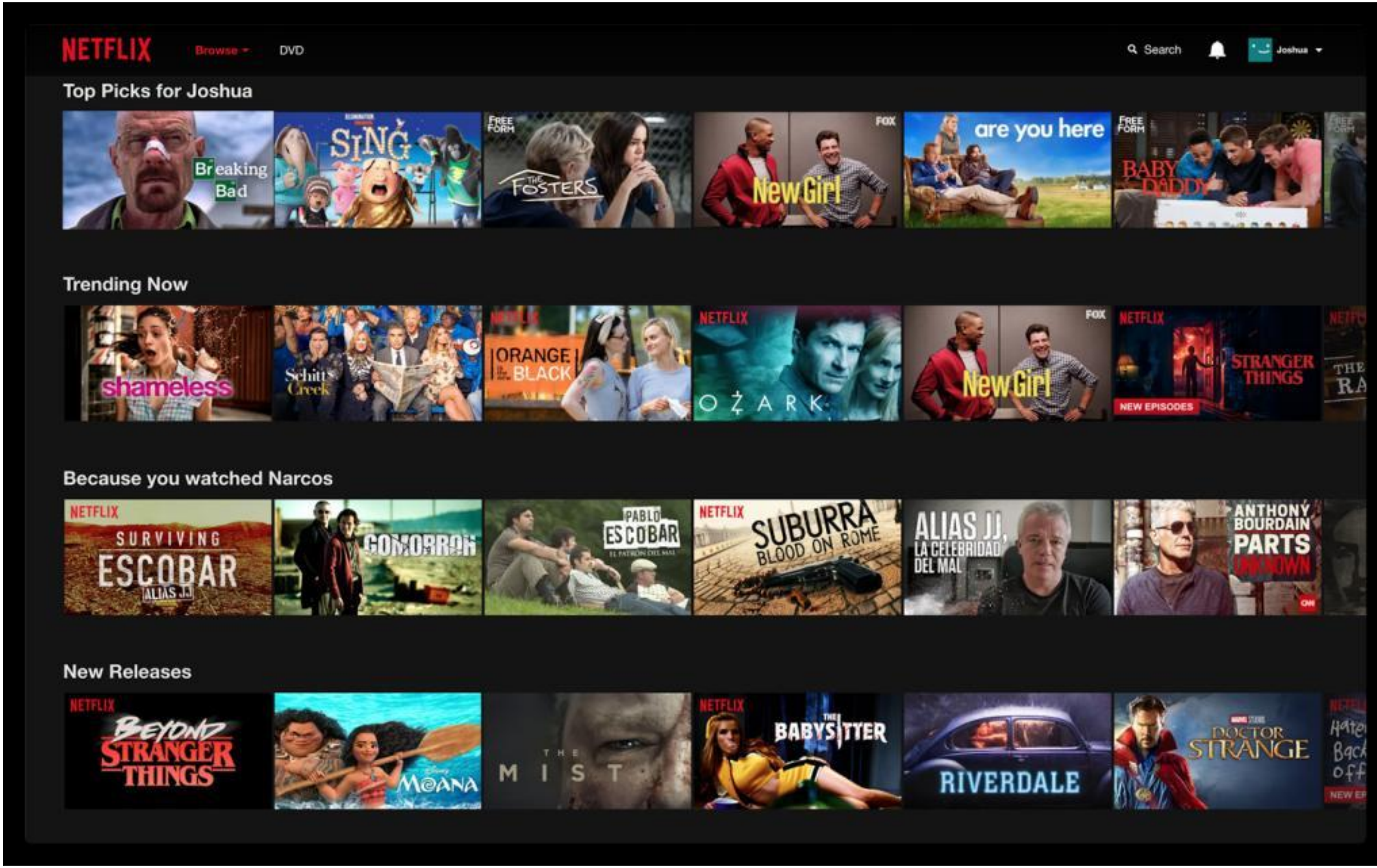
OPTIMIZATION IN EVERYDAY LIFE



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POWER SYSTEM OPTIMIZATION IN THE NEWS

Blog

It's Time for Utilities to Back Smart Grid Optimization with the Right Tech

https://www.powermag.com/blog/its-time-for-utilities-to-back-smart-grid-optimization-with-the-right-tech/

Office of
ENERGY EFFICIENCY & RENEWABLE ENERGY

WATER POWER TECHNOLOGIES OFFICE

Water Power Technologies Office

New Research Helps Optimize Hydropower for Energy and the Environment

MAY 22, 2024

https://www.energy.gov/eere/water/articles/new-research-helps-optimize-hydropower-energy-and-environment

Digital Power Optimization plans 100 MW of behind-the-meter datacenters in Texas

https://www.spglobal.com/marketintelligence/en/news-insights/latest-news-headlines/digital-power-optimization-plans-100-mw-of-behind-the-meter-datacenters-in-texas-80025625

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1. Introduction to Optimization

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CLASSIFICATION OF OPTIMIZATION PROBLEMS

Linear vs non-linear

Convex vs non-convex

Unconstrained vs constrained

Continuous vs discrete

Static vs dynamic

Deterministic vs stochastic

Single objective vs multiple objective

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THE STRUCTURE OF OPTIMIZATION PROBLEMS

Objective function: what is to be maximized or minimized

Maximize profit
Minimize cost

Decision variables: the decisions or choices that the decision maker can make

Output of a generation unit
Build a new generation unit
Retire an existing generation unit
Charge or discharge a battery

Constraints: limits on the decision variables

Maximum output of a generation unit
Maximum flow on a transmission line

Parameters: fixed numerical values that affect the objective function and constraints

Generation variable costs
Size of an energy storage unit

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TYPES OF CONSTRAINTS IN POWER SYSTEMS

Technical & Policy Constraints

- Generation
- Transmission
- Supply/demand
- Reliability
- Air emissions

Mathematical Constraints

- Equality, e.g., supply = demand
- Inequality, e.g., generation operation
- Non-negativity, e.g., minimum generation

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SOME ADDITIONAL TERMINOLOGY

- Optimal value – the minimum or maximum of the objective function over all feasible points
- Feasible points – any potential solutions that satisfy all of the constraints
- Solution – the values of the decision variables that minimize or maximize the objective function
- Active or binding constraint – a constraint that is limiting one or more of the decision variables
- Locally optimal – an optimal solution in a smaller set of feasible points
- Globally optimal - an optimal solution over the entire set of feasible points
- Unique solution – the only optimal solution

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SOLVING OPTIMIZATION PROBLEMS

- For many problems, off the shelf software packages with well-known algorithms can be used
- Standard computer software and languages have extensive optimization capabilities and packages (even Excel to a limited degree)
- Types of solution methods (algorithms)
  - Simulation – calculate many decision variables and pick the best one from the simulated sample (what if, heuristics)
  - Enumeration ("brute force") – calculate all possible decisions and select the best one
  - Greedy algorithm – take first best choice, then second, …
  - Specific solutions for specific types of problems, e.g., linear programing for linear optimization problems (may have to set tolerance levels, penalty terms, and time limits depending on the algorithm)

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SOLVING OPTIMIZATION PROBLEMS, con't

- Start small and simple
- Spend time getting a solid, clean data set
- Test for solvability
  - Depending on the algorithm, may need to set various "tuning factors"
- Conduct extensive sensitivity analyses to try to "break the model"
- Add more complexity
- Repeat

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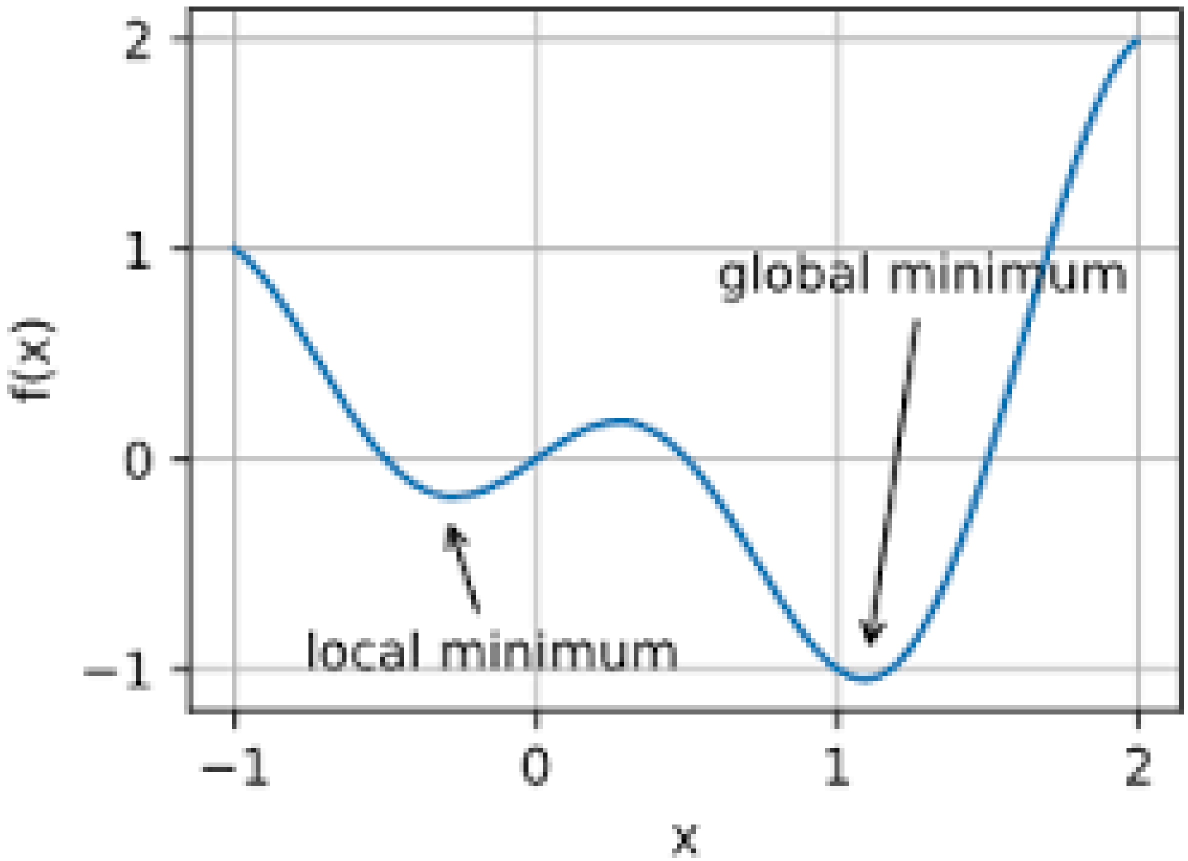
SIMPLIFIED GEOMETRY OF OPTIMIZATION



How do the first and second order conditions inform whether you are at a minimum, maximum, or inflection point?

http://d2l.ai/chapter_optimization/optimization-intro.html

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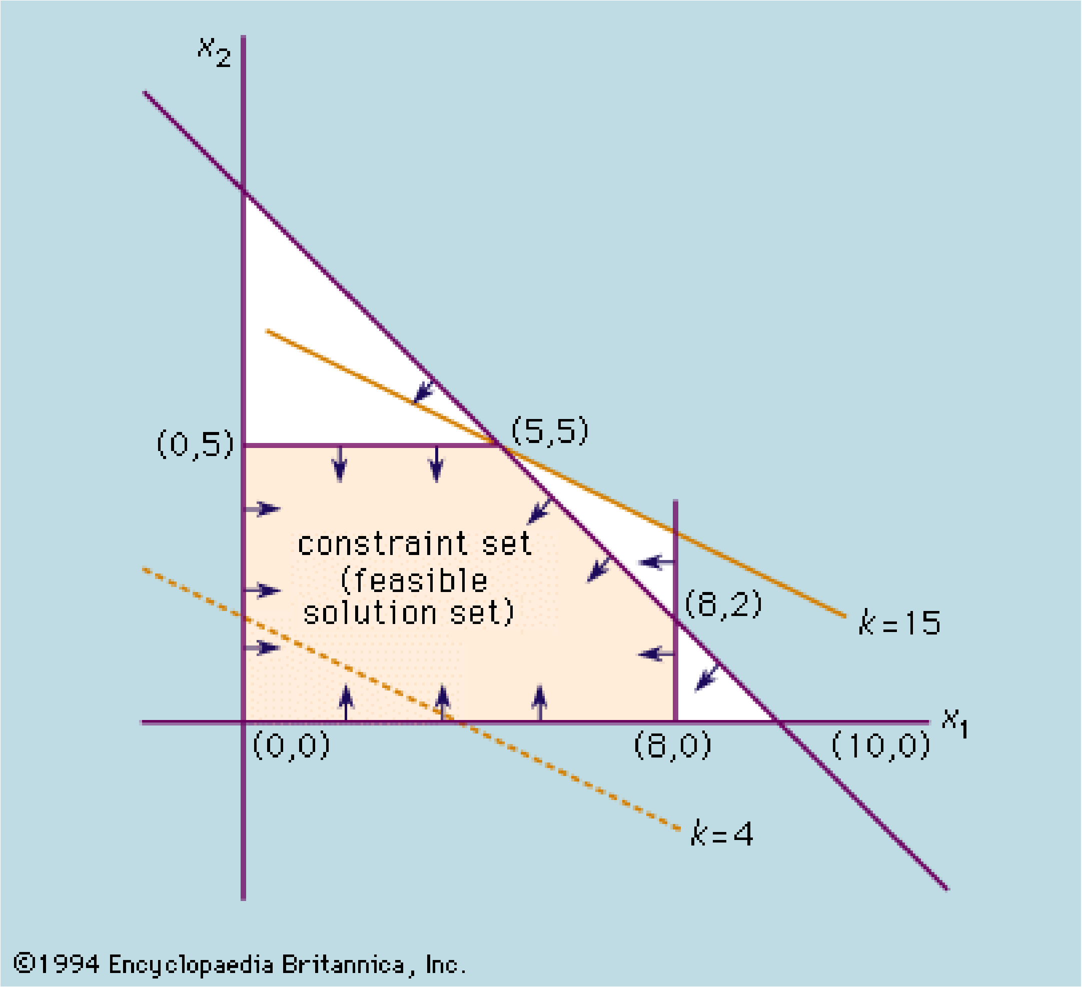
SIMPLIFIED GEOMETRY OF OPTIMIZATION – LINEAR PROGRM (LP)



https://www.britannica.com/science/optimization

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ECONOMIC DISPATCH

Words

- Minimize the cost of dispatching generation units subject to
  - Supply equals demand
  - Generation units are operated within their minimum and maximum capacities

Symbols

- Min $c^T x = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

  subject to
- $\sum x_i = x_1 + x_2 + \ldots + x_n = D$
- $0 \leq x_i \leq G_i$
- the parameter c_i is the variable cost of generation unit i
- the parameter D is total demand
- the decision variable x_i is the output of unit i
- the parameter G_i is the maximum capacity of generation unit I
- x* indicates the optimal solution
- f*(x*) indicates the optimal value of the objective function

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SOME GENERAL RESULTS

- All minimization problems can be converted to maximization problems by multiplying the objective function by a negative sign
- Adding a constraint to an optimization problem never improves the answer
  - At best it does not affect the answer
  - At worst it makes the answer less optimal than without the constraint
  - Similarly, tightening a constraint never improves the answer
  - **What happens when you add more decision variables?**
- Analysis is "on the margin"
- If there is sufficient competition, minimizing total cost is the same as maximizing generator profits

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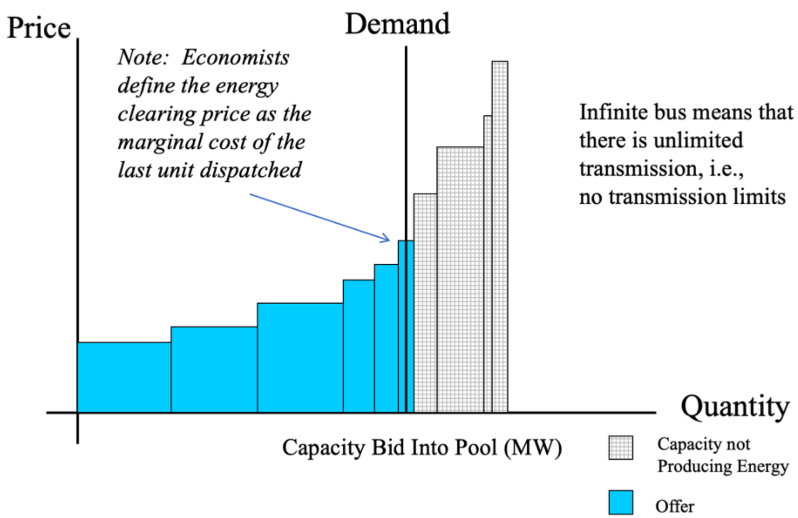
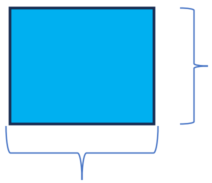
2. Economic Dispatch Case Study

ECONOMIC DISPATCH

Picture





Symbols

- Min $c^T x = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

  subject to

- $\sum x_i = x_1 + x_2 + \ldots + x_n = \text{Demand}$
- $0 \leq x_i \leq G_i$
- where the parameter c_i is the variable cost of generation unit i
- where the parameter G_i is the maximum capacity of generation unit I
- where the decision variable x_i is the output of unit i

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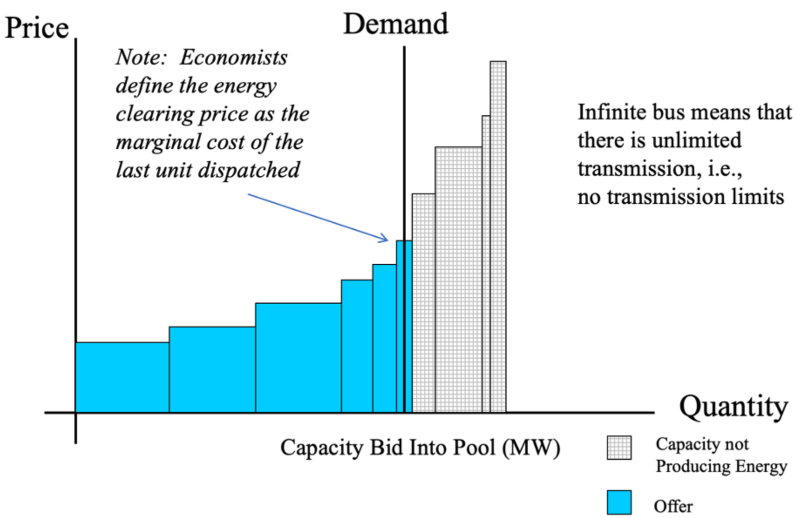
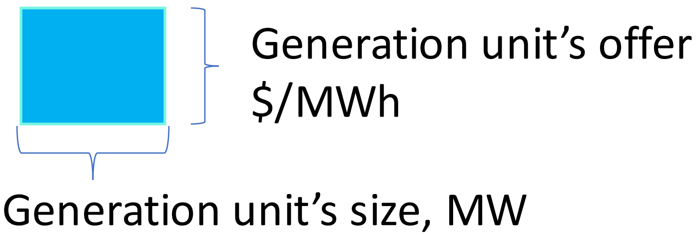
ECONOMIC DISPATCH NO TRANSMISSION CONSTRAINTS

Picture





What is missing?

- Generation ramping constraints
- Generation startup and shutdown constraints
- Generation minimum downtime and runtime constraints
- Transmission constraints

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ECONOMIC DISPATCH NO TRANSMISSION CONSTRAINTS

	G_i	c_i
Generation	**Size (MW)**	**Offer ($/MWh)**
A	50	72
B	400	32
C	800	10
D	300	37
E	50	60
F	450	28
G	600	15
H	50	65
I	500	20
J	250	42
K	400	35
L	250	40

→

Generation	Size (M)	Offer ($/MWh)	Cummulative Output
C	800	10	800
G	600	15	1400
I	500	20	1900
F	450	28	2350
B	400	32	2750
K	400	35	3150
D	300	37	3450
L	250	40	3700
J	250	42	3950
E	50	60	4000
H	50	65	4050
A	50	72	4100

Sort Offers
Low to High

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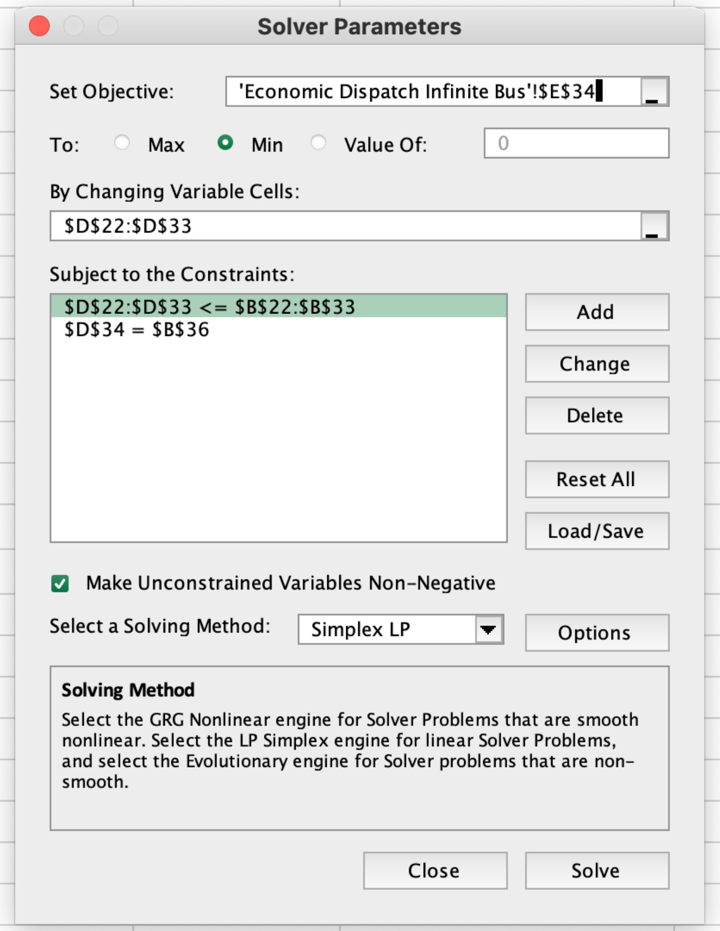
EXCEL SOLVER: SUFFICIENT SUPPLY TO MEET DEMAND

	G_i	c_i	x_i	
Generation	**Size (MW)**	**Offer ($/MWh)**	**Output**	**Cost**
A	50	72	0	$ -
B	400	32	150	$ 4,800
C	800	10	800	$ 8,000
D	300	37	0	$ -
E	50	60	0	$ -
F	450	28	450	$ 12,600
G	600	15	600	$ 9,000
H	50	65	0	$ -
I	500	20	500	$ 10,000
J	250	42	0	$ -
K	400	35	0	$ -
L	250	40	0	$ -
			2500	$ 44,400
Demand	2500			



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SENSITIVITY ANALYSIS

Microsoft Excel 16.88 Sensitivity Report
Worksheet: [Fundamentals of Pwr Sys Opt.xlsx]Economic Dispatch Infinite Bus
Report Created: 9/15/24 8:29:05 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D22	A Output	0	40	72	1E+30	40
D23	B Output	150	0	32	3	4
D24	C Output	800	-22	10	22	1E+30
D25	D Output	0	5	37	1E+30	5
D26	E Output	0	28	60	1E+30	28
D27	F Output	450	-4	28	4	1E+30
D28	G Output	600	-17	15	17	1E+30
D29	H Output	0	33	65	1E+30	33
D30	I Output	500	-12	20	12	1E+30
D31	J Output	0	10	42	1E+30	10
D32	K Output	0	3	35	1E+30	3
D33	L Output	0	8	40	1E+30	8

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D34	Output	2500	32	2500	250	150

Generation	Size (MW)	Offer ($/MWh)	Output	Cost
A	50	72	0	$ -
B	400	32	150	$ 4,800
C	800	10	800	$ 8,000
D	300	37	0	$ -
E	50	60	0	$ -
F	450	28	450	$ 12,600
G	600	15	600	$ 9,000
H	50	65	0	$ -
I	500	20	500	$ 10,000
J	250	42	0	$ -
K	400	35	0	$ -
L	250	40	0	$ -
			2500	$ 44,400
Demand	2500			

<u>Shadow price</u>: change in the optimal value of the objective function for a 1 unit change in a constraint

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ECONOMIC DISPATCH NO TRANSMISSION CONSTRAINTS WITH CO_2 PRICE

Generation	Size (MW)	Offer ($/MWh)
A	50	72
B	400	32
C	800	10
D	300	37
E	50	60
F	450	28
G	600	15
H	50	65
I	500	20
J	250	42
K	400	35
L	250	40

→

Generation	Size (M)	Offer ($/MWh)	Cummulative Output
C	800	10	800
G	600	15	1400
I	500	20	1900
F	450	28	2350
B	400	32	2750
K	400	35	3150
D	300	37	3450
L	250	40	3700
J	250	42	3950
E	50	60	4000
H	50	65	4050
A	50	72	4100
		Sort Offers Low to High	

How would you incorporate a CO_2 emission allowance cost?

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ECONOMIC DISPATCH NO TRANSMISSION CONSTRAINTS WITH CO_2 CAP

Generation	Size (MW)	Offer ($/MWh)
A	50	72
B	400	32
C	800	10
D	300	37
E	50	60
F	450	28
G	600	15
H	50	65
I	500	20
J	250	42
K	400	35
L	250	40

→

Generation	Size (M)	Offer ($/MWh)	Cummulative Output
C	800	10	800
G	600	15	1400
I	500	20	1900
F	450	28	2350
B	400	32	2750
K	400	35	3150
D	300	37	3450
L	250	40	3700
J	250	42	3950
E	50	60	4000
H	50	65	4050
A	50	72	4100
		Sort Offers Low to High	

How would you incorporate a CO_2 emission cap?

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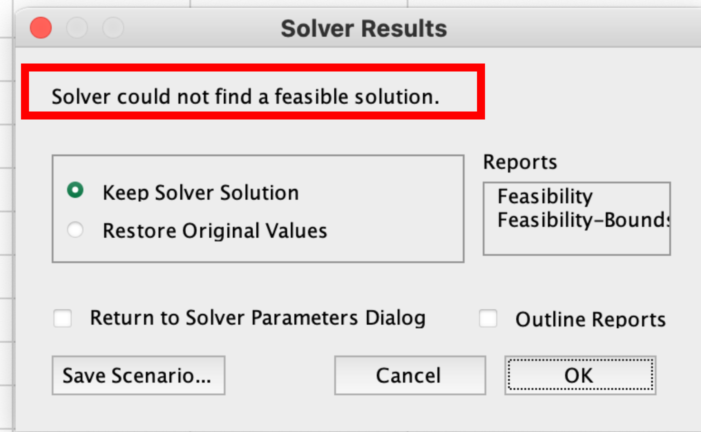
EXCEL SOLVER: DEMAND EXCEEDS SUPPLY

Generation	Size (MW)	Offer ($/MWh)	Output	Cost
A	50	72	0	$ -
B	400	32	0	$ -
C	800	10	0	$ -
D	300	37	0	$ -
E	50	60	0	$ -
F	450	28	0	$ -
G	600	15	0	$ -
H	50	65	0	$ -
I	500	20	0	$ -
J	250	42	0	$ -
K	400	35	0	$ -
L	250	40	0	$ -
	4,100		-	$ -
Demand	4,500			



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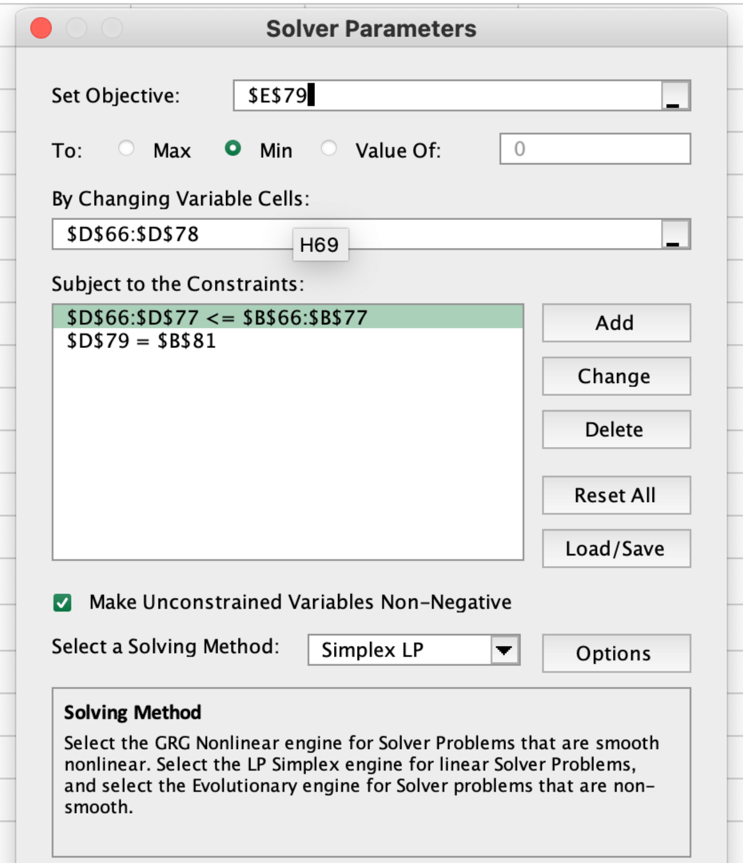
EXCEL SOLVER: DEMAND EXCEEDS SUPPLY, VALUE OF LOST LOAD

Generation	Size (MW)	Offer ($/MWh)	Output	Cost
A	50	72	50	$ 3,600
B	400	32	400	$ 12,800
C	800	10	800	$ 8,000
D	300	37	300	$ 11,100
E	50	60	50	$ 3,000
F	450	28	450	$ 12,600
G	600	15	600	$ 9,000
H	50	65	50	$ 3,250
I	500	20	500	$ 10,000
J	250	42	250	$ 10,500
K	400	35	400	$ 14,000
L	250	40	250	$ 10,000
Unserved Energy	unlimited	10,000	400	$ 4,000,000
			4,500	$ 4,107,850
Demand	4,500			



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VALUE OF LOST LOAD (VOLL)

Definition

The Value of Lost Load (VOLL) is an indicator of the economic value that consumers place on the energy not served in case of a supply disruption, e.g. an electricity outage (blackout). VOLL is broadly used by industry and regulators for benchmarking the operating conditions of an energy system.

Interpretations

- Literal cost to consumers
- An oversimplification of individual consumer demand curves
- An approximate value used for planning purposes

Ways to Measure VOLL

- Surveys
- Costs of behaviors in response to actual or anticipated power outages
- Macroeconomic studies
- Estimates of cost of lost production

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EXCEL SOLVER: DEMAND RESPONSE

Generation	Size (MW)	Offer ($/MWh)	Output	Cost
A	50	72	50	$ 3,600
B	400	32	400	$ 12,800
C	800	10	800	$ 8,000
D	300	37	300	$ 11,100
E	50	60	50	$ 3,000
F	450	28	450	$ 12,600
G	600	15	600	$ 9,000
H	50	65	50	$ 3,250
I	500	20	500	$ 10,000
J	250	42	250	$ 10,500
K	400	35	400	$ 14,000
L	250	40	250	$ 10,000
Demand Response 1	100	100	100	$ 10,000
Demand Response 2	200	150	200	$ 30,000
Demand Response 3	150	200	100	$ 20,000
	4,550		4500	$ 167,850
Demand	4,500			

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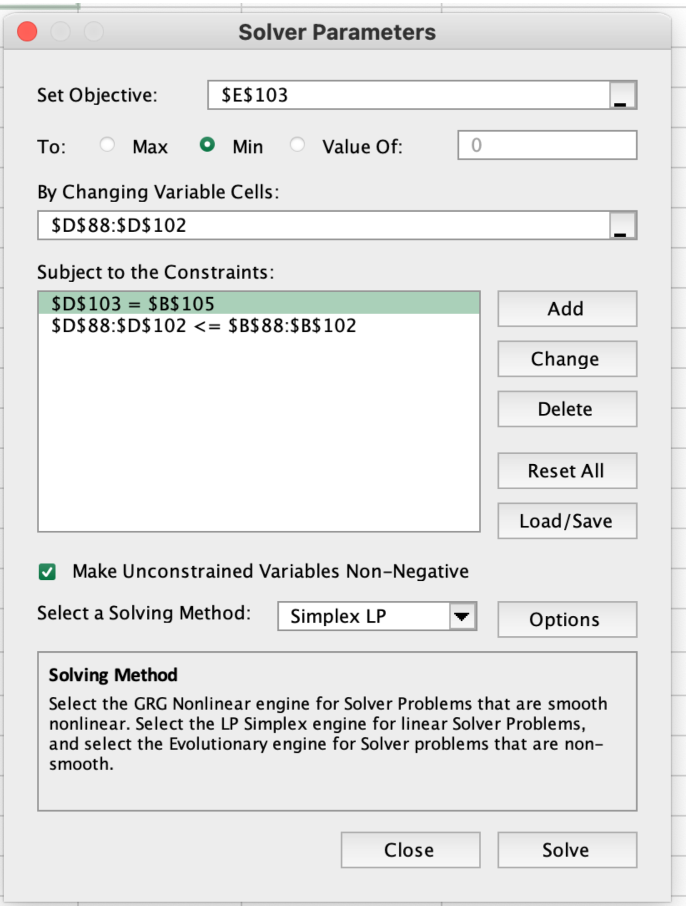

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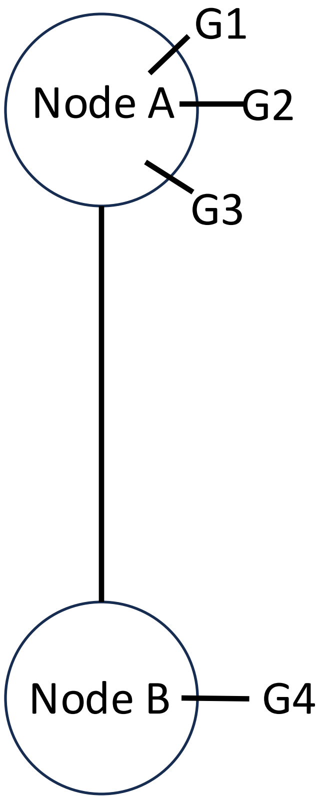
ECONOMIC DISPATCH WITH TRANSMISSION CONSTRAINTS

Picture



Generation and Transmission Assumptions

Generation, Transmission, Load	Assumption
G1	Offer $10/MWH 100 MW
G2	Offer $20/MWH 100 MW
G3	Offer $30/MWH 100 MW
G4	Offer $40/MWH 100 MW
Transmission Line	200 MW
Load	250 MW

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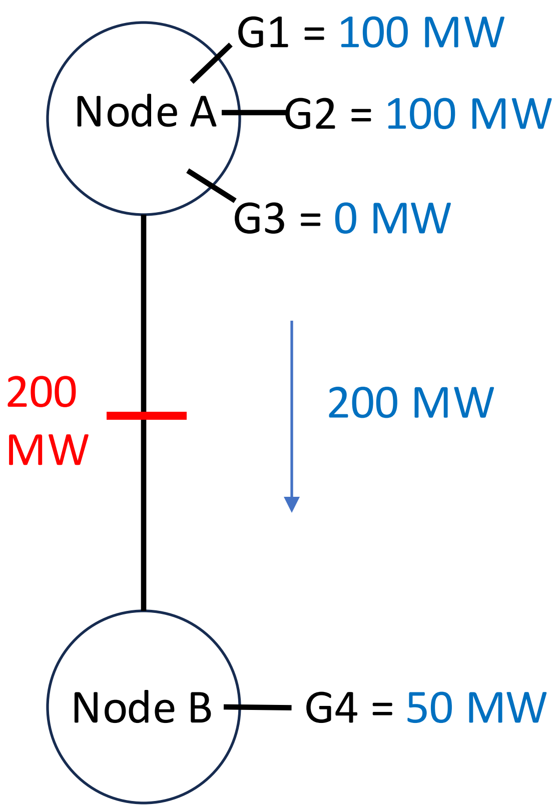
ECONOMIC DISPATCH WITH TRANSMISSION CONSTRAINTS - SOLUTION

Picture



Generation and Transmission Assumptions

Generation, Transmission, Load	Assumption
G1	Offer $10/MWH 100 MW
G2	Offer $20/MWH 100 MW
G3	Offer $30/MWH 100 MW
G4	Offer $40/MWH 100 MW
Transmission Line Constraint	200 MW
Load	250 MW

Total cost = $10*100 + $20* 100 + $40*50 = $5000

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ECONOMIC DISPATCH LOOP FLOW PROBLEM

No Transmission Constraints

3 Bus Example

- Lines have identical impedances
- No transmission losses

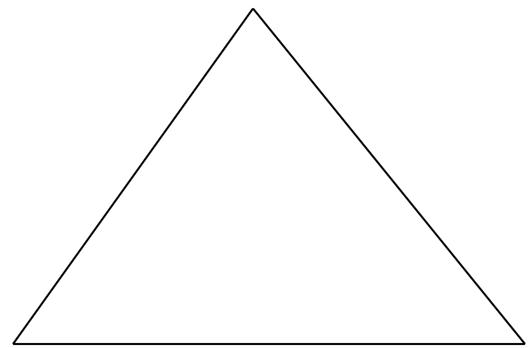
What is the least-cost dispatch?

- Without any transmission constraint?

=> Solution

- G1: Dispatched 0 MW
- G2: Dispatched 150 MW

G1 Max Output = 90 MW at $50/MWh



G2 Max Output = 150 MW at $30/MWh

Load= 150 MW

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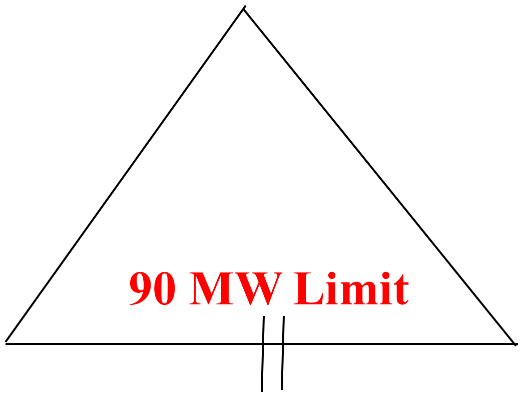
ECONOMIC DISPATCH LOOP FLOW PROBLEM

Transmission Constraint

G1 = 90 MW capacity at $50/MWh



3 Bus Example

- Lines have identical impedances
- No transmission losses

G2 = 150 MW capacity at $30/MWh

Load= 150 MW

What is the least-cost dispatch?

- With a 90 MW transmission constraint

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ECONOMIC DISPATCH LOOP FLOW EQUATIONS

Min $50G_1 + 30G_2$

Subject to

$G_1 + G_2 = 150$

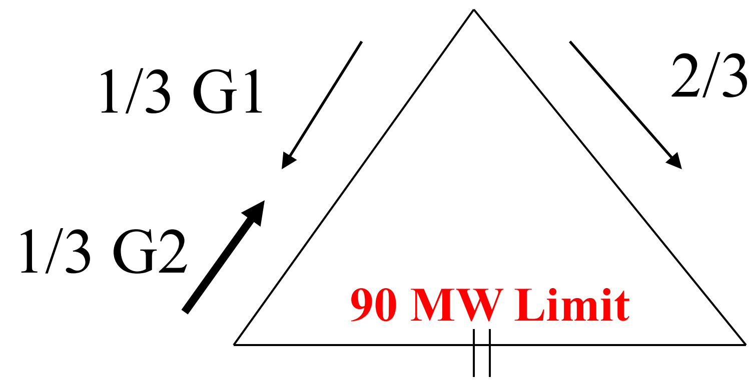
$1/3G_1 + 2/3G_2 \leq 90$

$G_1 \leq 90$

$G_2 \leq 150$

$G_1, G_2 \geq 0$

G1 = 90 MW max capacity at $50/MWh



G2 = 150 max Capacity MW At $30/MWh

L= 150 MW

→ 1/3 G1=10 MW

→ 2/3 G2=80 MW

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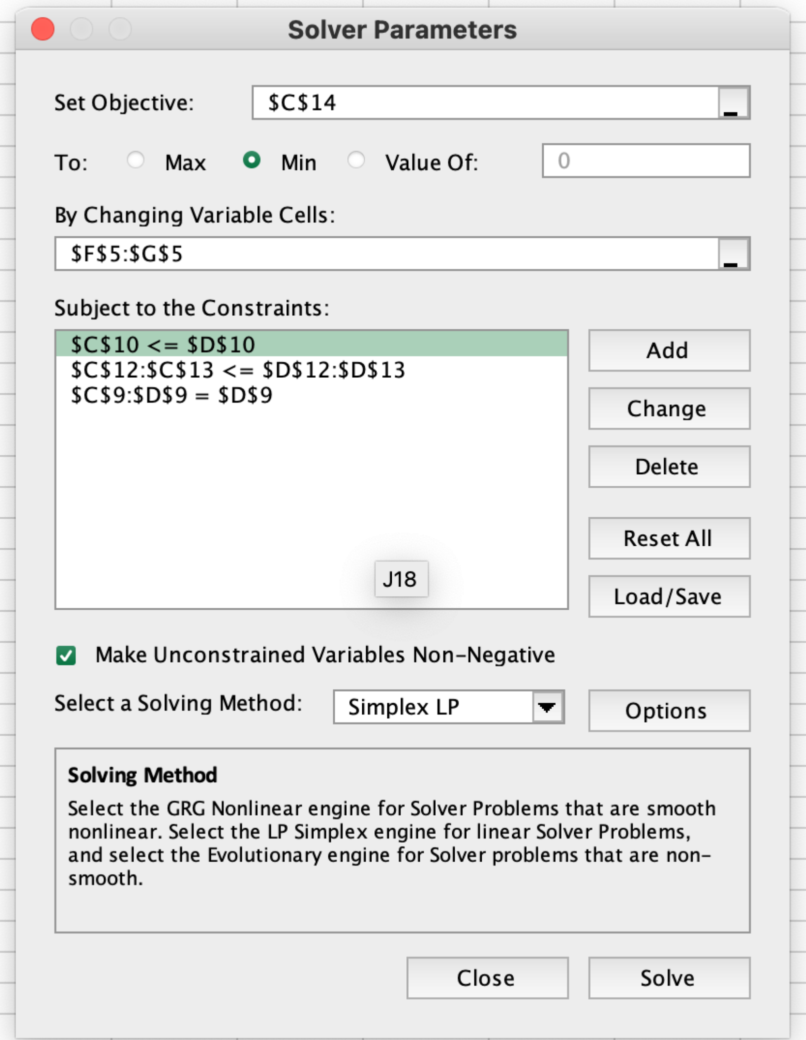
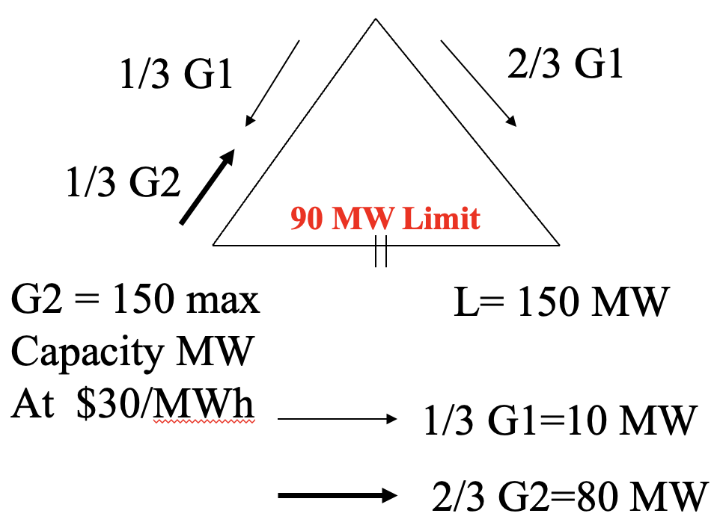
EXCEL SOLVER, 3 BUS LOOP FLOW

Economic Dispatch, 3 Bus Loop Flow

				Decision Varables	
				G1	G2
minimize	$ 5,100			30	120
subject to					
	Value	Limit			
G1+G2 =	150	150			
1/3G1+2/3G2<=	90	90			
G1<=	30	90			
G2<=	120	150			
G1 >=	30	30			
G2>=	120	120			



G1 = 90 MW max capacity at $50/MWh



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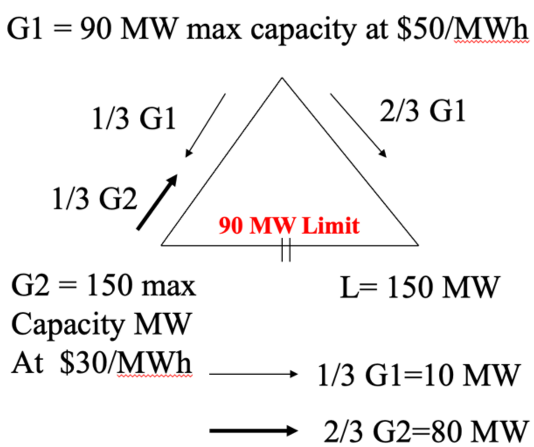
EXCEL SOLVER, 3 BUS, TRANSMISSION EXPANSION

Case 1			
	No. Hours Unconstrained		5,000
	No. Hours Constrained		3,760
	Note: 8760 hours in a non-leap year		
Congestion cost per hour when constrained		$	600
Cost of Congestion per year		$	2,256,000
Net Present Value of Congestion Cost		$	21,267,119
	No. Years		30
	Discount Rate		10%

Case 2			
	No. Hours Unconstrained		-
	No. Hours Constrained		8,760
	Note: 8760 hours in a non-leap year		
Congestion cost per hour when constraine		$	600
Cost of Congestion per year		$	5,256,000
Net Present Value of Congestion Cost			$49,547,862
	No. Years		30
	Discount Rate		10%



How much should society be willing to pay for transmission that eliminates the binding transmission constraint?

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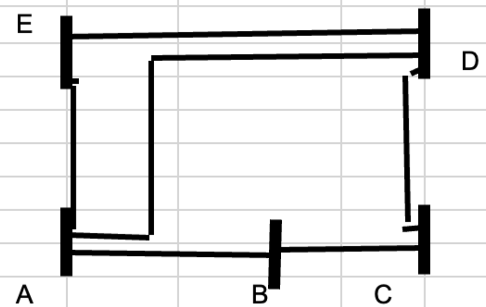
EXCEL SOLVER, 5 BUS & DEMAND BIDS

Location	Offers Qty.	Offers Price	Offer Choice	Q*$
A	10	5	10	50
A	20	15	20	300
B	15	6	15	90
B	40	7	40	280
C	30	6	30	180
C	40	15	40	600
C	20	25	0	0
D	10	10	10	100
D	20	15	20	300
D	60	30	0	0
D	75	18	0	0
		sum	185	1900



P Injection	
A	-40
B	5
C	70
D	-10
E	-25

Index
4275

Location	Demand bids Q	Demand bids $	Demand Choice	Q * $
E	20	35	20	700
E	15	15	5	75
A	25	40	25	1000
A	45	30	45	1350
B	50	25	50	1250
C	60	5	0	0
D	40	45	40	1800
		sum	185	6175

SF = Shift Factors

SF Matrix							Branch	Line flows SF * P inj		
0	-0.66942	-0.542789	-0.19457	-0.0344	-100	>=	a-b	-38.535354	<=	100
0	-0.18002	-0.249214	-0.4395	-0.0778	-100	>=	a-d	-12.004771	<=	100
0	-0.15142	-0.209619	-0.36967	-0.8871	-100	>=	a-e	10.4433951	<=	100
0	0.329811	-0.543415	-0.1948	-0.0345	-100	>=	b-c	-33.579798	<=	30
0	0.329811	0.456585	-0.1948	-0.0345	-100	>=	c-d	36.4202018	<=	100
0	0.151659	0.209954	0.370262	-0.1115	-100	>=	d-e	14.5398952	<=	100

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Rau, 2003, p. 95

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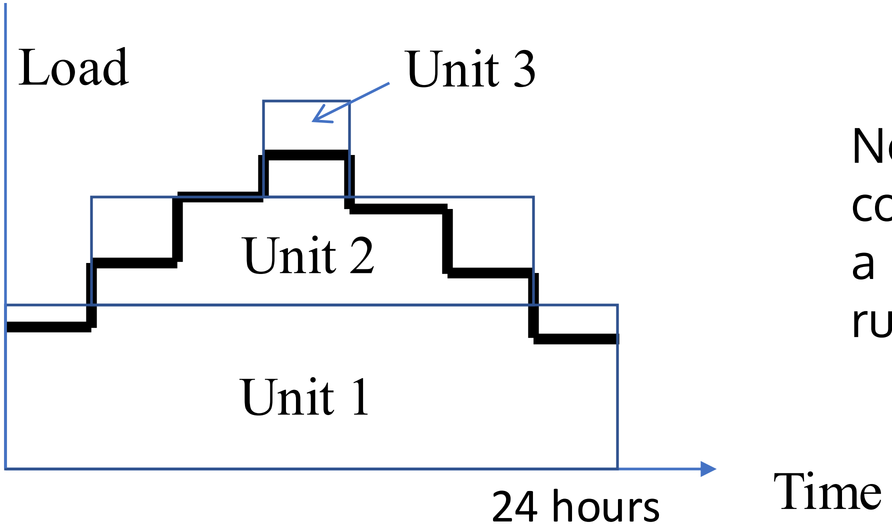
UNIT COMMITMENT

Unit Commitment: Minimize start-up, no-load, and running costs subject to meeting load reliably & the constraints of generation units (size, start-up time, ramp rates, min. run times, min. down times, etc.) over 24 hours.

The solution to this optimization is not only the output level of individual generation units but which units to commit.

Dispatch: Minimize no-load and running costs subject to meeting load reliably & the constraints of the generation units (size and ramp rates) over 5 minutes.

The solution to this optimization problem is the output level of individual generation units; solved every 5 minutes.



Note: Unit commitment is much more complicated than dispatching and requires a more sophisticated algorithm and longer run time.

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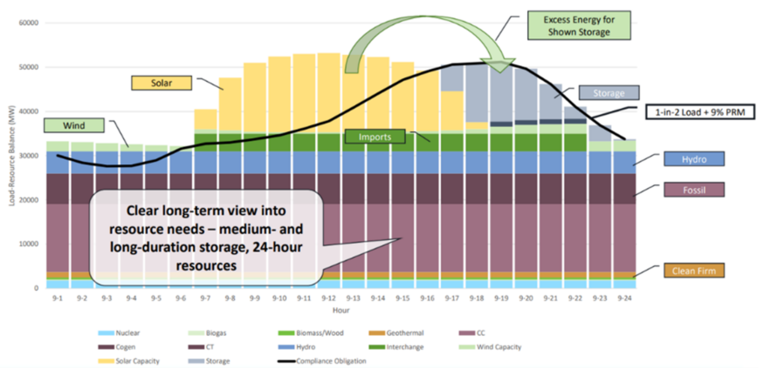
UNIT COMMITMENT NO TRANSMISSION CONSTRAINTS

Picture



What is missing?

- Generation ramping constraints
- Generation startup and shutdown constraints
- Generation minimum downtime and runtime constraints
- Transmission constraints

SECURITY CONSTRAINED ECONOMIC DISPATCH AND UNIT COMMITMENT

Security constrained means that the failure of any one generation or transmission component will not result in the disconnection of firm load

N-1 security constraint

SCED and SCUC are the economic dispatch and the unit commitment problems with the added constraints to the problem to implement N-1

Hajiesmaili, Mohammad H., Desmond Cai, and Enrique Mallada. "Understanding the inefficiency of security-constrained economic dispatch." *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*. IEEE, 2017.

The ED problem minimizes generation costs subject to operating constraints and is given by:

$$\begin{aligned} \text{ED} : \min_{\mathbf{q}} \quad & \boldsymbol{\alpha}^{\mathsf{T}}\mathbf{q} && \text{(2a)} \\ \text{s.t.} \quad & \mathbf{0} \le \mathbf{q} \le \overline{\mathbf{q}}, && \text{(2b)} \\ & \mathbf{1}^{\mathsf{T}}(\mathbf{q}-\mathbf{d}) = 0, && \text{(2c)} \\ & -\overline{\mathbf{f}} \le \mathbf{H}(\mathbf{q}-\mathbf{d}) \le \overline{\mathbf{f}}. && \text{(2d)} \end{aligned}$$

Constraint (2b) restricts generations to capacities, constraint (2c) enforces supply-demand balance, and constraint (2d) restricts line flows to line limits.

By focusing on robustness to the outage of any single line, we formulate the SCED problem. Associate with the outage of an edge $e \in \mathcal{E}$, an $m-1$ vector $\overline{\mathbf{f}}_{-e} = (f_{e'} : e' \in \mathcal{E}, e' \neq e)$ of line capacities and $(m-1) \times n$ matrix $\mathbf{H}_{-e}$ of shift factors. We are interested in the following SCED problem:

$$\begin{aligned} \text{SCED} : \min_{\mathbf{q}} \quad & \boldsymbol{\alpha}^{\mathsf{T}}\mathbf{q} && \text{(3a)} \\ \text{s.t.} \quad & \mathbf{0} \le \mathbf{q} \le \overline{\mathbf{q}}, && \text{(3b)} \\ & \mathbf{1}^{\mathsf{T}}(\mathbf{q}-\mathbf{d}) = 0, && \text{(3c)} \\ & -\overline{\mathbf{f}} \le \mathbf{H}(\mathbf{q}-\mathbf{d}) \le \overline{\mathbf{f}}, && \text{(3d)} \\ & -\overline{\mathbf{f}}_{-e} \le \mathbf{H}_{-e}(\mathbf{q}-\mathbf{d}) \le \overline{\mathbf{f}}_{-e}, \forall e. && \text{(3e)} \end{aligned}$$

Note that SCED contains $2m(m-1)$ more constraints than ED, which are represented by (3e), each of which is associated with a unique line outage.

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3. Data Center Case Study

Google and Microsoft back 24/7 carbon-free energy marketplace

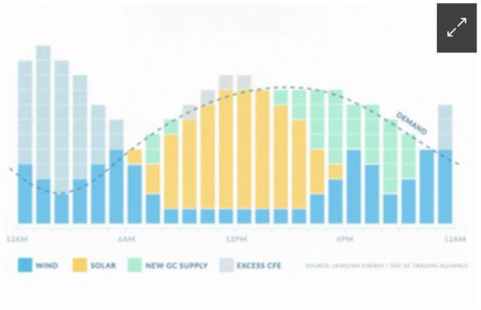
LevelTen Energy sets up GC Trading Alliance

December 19, 2023 By: Peter Judge Have your say

Renewables transaction company LevelTen Energy has set up a marketplace for time-based energy certificates, aimed at supporting 24/7 renewable energy commitments.

The GC Trading Alliance, supported by Google, Microsoft, and others, will be a platform to exchange "granular certificates" that verify the time and place where carbon-free energy is generated.

The Alliance's platform is being set up with the Intercontinental Exchange (ICE), a US financial exchange operation company.



LevelTen graph illustrates the mismatch between demand, and carbon free capacity – LevelTen Energy

Channeling investment to clean energy

"To say this is an exciting moment is an understatement," said LevelTen CEO Bryce Smith in a blog. "This platform will provide market signals that will optimize the dispatch of existing carbon-free generation, and fuel investments in new assets that current market signals do not reach."

The move is designed so energy buyers can source carbon-free energy around the clock, and energy sellers are incentivized to provide that energy, either through new generation or storage of energy generated at times of low demand.

Alliance members, including AES, Constellation, Google, and Microsoft, will be the first customers on the platform, and hope it can be scaled to accelerate energy transition.

Hyperscale data center operators have been among the leaders in buying clean energy, and have boosted the markets for Renewable Energy Certificates (RECs) and Power Purchase Agreements (PPAs) which pay for energy generation.

Recall that that the impact of GHG gases depends on the cumulative emissions, not when they are emitted (subject to some mild caveats).

What does this fact suggest about carbon-free hourly matching?

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https://www.datacenterdynamics.com/en/news/google-and-microsoft-back-247-carbon-free-energy-marketplace/

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DATA CENTER CASE STUDY - MOTIVATION

Google Data Centers

Locations Innovations Data and Security Efficiency 24/7 Carbon-Free Energy Gallery Life@ Podcast Discover FAQ

24/7 Carbon-Free Energy by 2030

From 2010 to 2023, we signed more than 115 agreements totaling over 14 GW of clean energy generation capacity—the equivalent of more than 36 million solar panels. Now, in our third decade of climate action, we've set a goal to run on 24/7 carbon-free energy on every grid where we operate by 2030, aiming to procure clean energy to meet our electricity needs, every hour of every day. Achieving this will also increase the impact of our clean energy procurement on the decarbonization of the grids that serve us.

A sustainability moonshot

In 2023 – for the seventh consecutive year – Google matched 100 percent of its global annual electricity consumption with purchases of renewable energy.

However, because of differences in the availability of renewable energy sources like solar and wind across the regions where we operate—and because of the variable supply of these resources—we still need to rely on carbon-emitting energy sources that power local grids.

That's why, in 2020, we set a goal to run on 24/7 carbon-free energy (CFE) on every grid where we operate by 2030, aiming to procure clean energy to meet our electricity needs, every hour of every day. Achieving this will also increase the impact of our clean energy procurement on the decarbonization of the grids that serve us.

Achieving 24/7 CFE is far more complex and technically challenging than annually matching our energy use with renewable energy purchases. No company of our size has achieved 24/7 CFE before, and there's no playbook for making it happen. But we see our efforts as part of a bigger picture: scaling new, global solutions for clean energy. We're excited to see others—like the U.S. federal government—set similar goals as well.

Our 24/7 CFE efforts are designed to maximize our contribution to the decarbonization of power grids worldwide. That's why we're supporting others to join us on the journey to 24/7 CFE, by sharing insights and lessons we're learning and new approaches we're developing. One such example is the United Nations 24/7 Carbon-Free Energy Compact, which we helped launch with Sustainable Energy for All and other partners in 2021.

LEARN MORE ABOUT HOW WE'RE AIMING TO ACHIEVE AROUND-THE-CLOCK CARBON-FREE ENERGY

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https://www.google.com/about/datacenters/cleanenergy/

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DATA CENTER CASE STUDY - MOTIVATION

Hourly carbon-free energy performance at an example data center

While Google buys large amounts of wind and solar power (symbolized by green spikes below), these resources are variable, meaning that our data centers still sometimes rely on carbon-based resources.

https://www.gstatic.com/gumdrop/sustainability/247-carbon-free-energy.pdf

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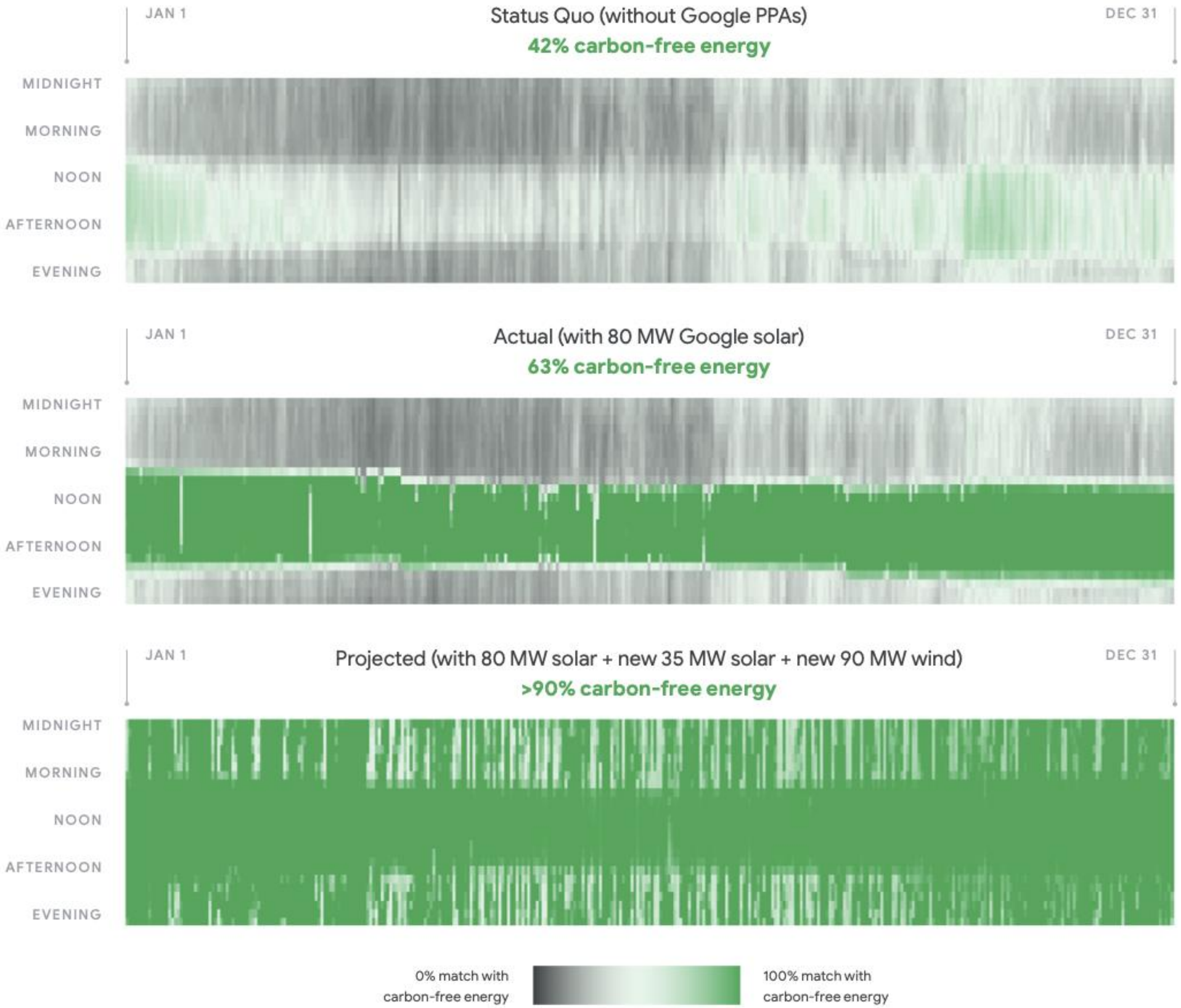
DATA CENTER CASE STUDY - MOTIVATION



https://www.gstatic.com/gumdrop/sustainability/247-carbon-free-energy.pdf

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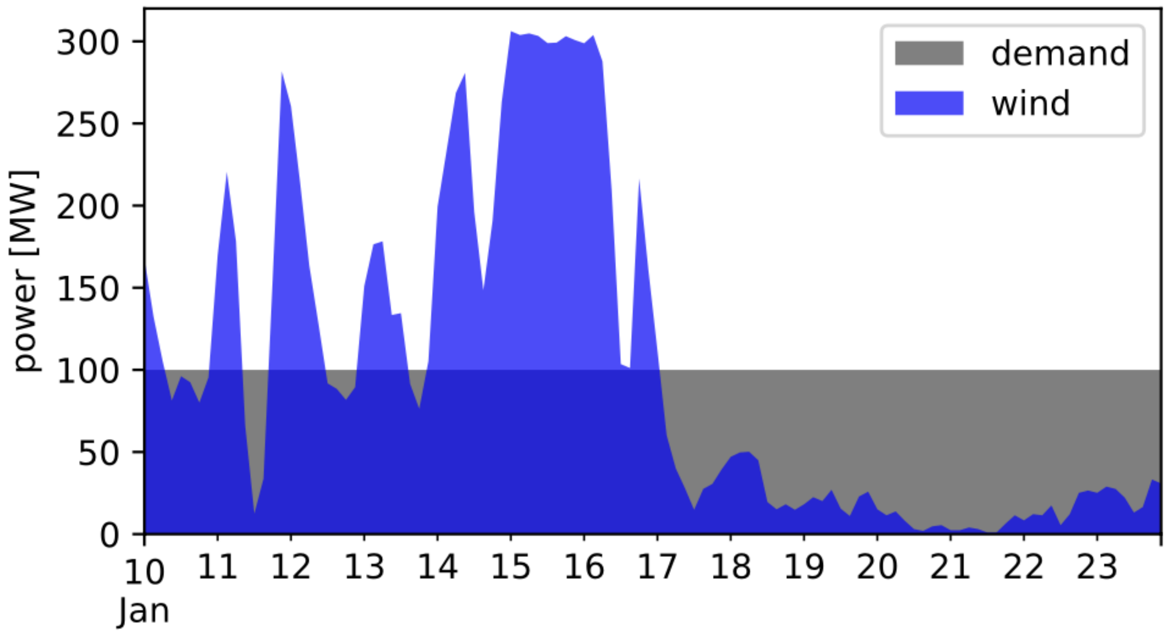
DATA CENTER CASE STUDY - MOTIVATION



- 100% RES PPAs result in periods of **oversupply** and **deficit**
- Hours of deficit must be met by **rest of system**
- These hours may have **high emissions** and **high prices**
- 24/7 carbon-free energy (CFE) matches demand on **hourly basis**

Why not just purchase CO_2 emission allowances?

Brown, Tom and Iegor Riepin, Modeling 24/7 Carbon-Free Procurement in Europe, TU Berlin presentation, June 15, 2022, p.2

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DATA CENTER CASE STUDY – PROBLEM FORMULATION

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

- What are possible objective functions?
- What are the possible decision variables?
- What are the constraints on each decision variable?
- What data would you need to conduct your analysis?

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DATA CENTER CASE STUDY – BOUNDARY ISSUES

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

- What is meant by the 100 MW data center
  - Electricity to run the data center during operations, maintenance, construction, decommissioning
  - Electricity for ancillary services, such as office space, parking lot lighting, etc.
  - Electricity on the grid or delivered to the data center
  - Equipment life cycle emissions
  - Emissions associated with employee and vendor transportation?

=> Need to precisely define the problem's boundaries

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DATA CENTER CASE STUDY – OBJECTIVE FUNCTION

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

Under what assumptions does the Brookfield profit maximizing objective function provide the same answer as the Brookfield cost minimization objective function?

What is the Data Center's objective function? When does it coincide with Brookfield's objective function?

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DATA CENTER CASE STUDY – OBJECTIVE FUNCTION

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

- TO BE COMPLETED BY PARTICIPANTS

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DATA CENTER CASE STUDY – DECISION VARIABLES

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

- TO BE COMPLETED BY PARTICIPANTS

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DATA CENTER CASE STUDY – OPTION TO DISPATCH THE DATA CENTER

Aligning compute load with carbon-free energy

Google's carbon-intelligent computing platform shifts flexible loads to times when wind and solar are abundant on the grid.

https://www.gstatic.com/gumdrop/sustainability/247-carbon-free-energy.pdf

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DATA CENTER CASE STUDY – CONSTRAINTS

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

- TO BE COMPLETED BY PARTICIPANTS

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DATA CENTER CASE STUDY – PROBLEM FORMULATION

Assume you have been asked to design a 24/7 carbon-free supply portfolio for a 100 MW data center and formulate this as an optimization problem.

Objective function:

min cost of self-generation + energy storage + market purchases/sales + data center data processing load

Constraints:

hourly data center demand = hourly CFE self-supply + market purchases – sales

self-generation is feasible

energy storage charging & discharging feasible

non-negativity constraints

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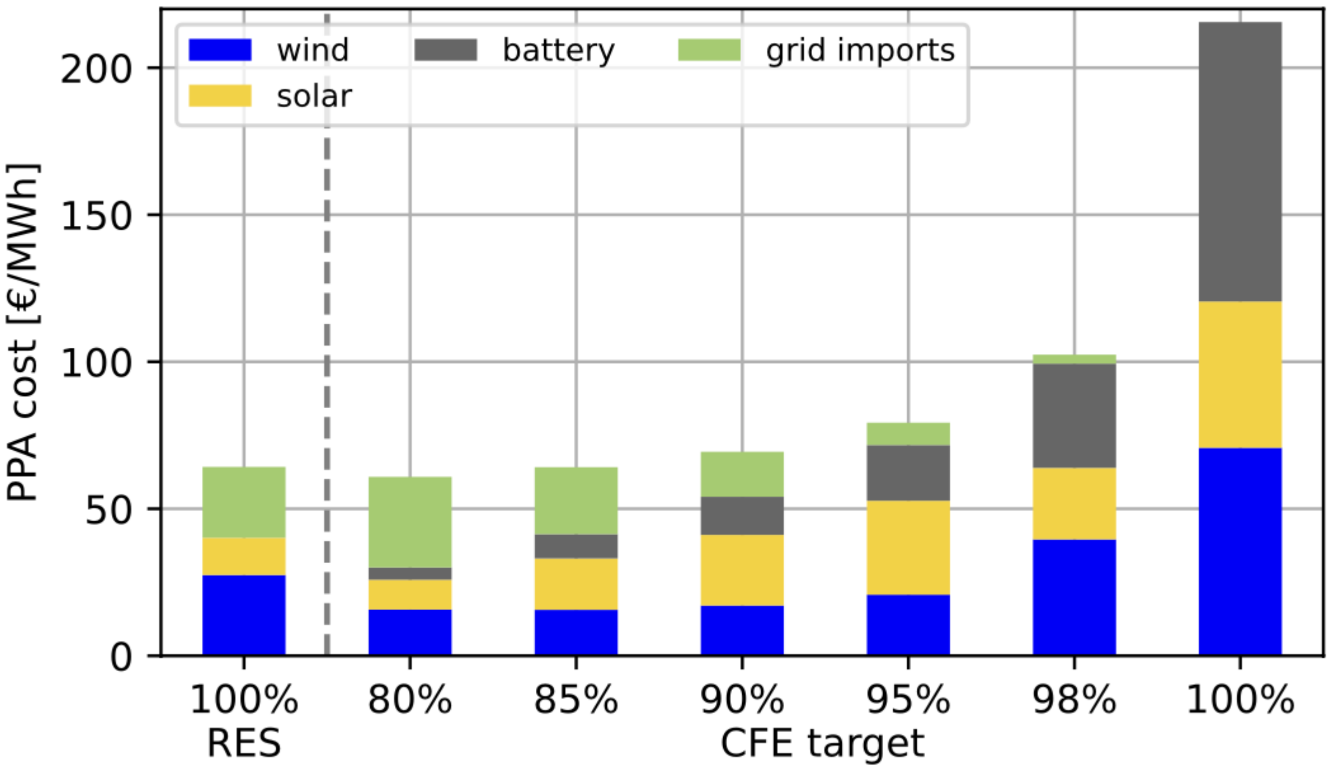
DATA CENTER CASE STUDY – RESULTS



- 100% RES builds wind and solar
- Higher CFE targets **include battery storage**
- 98% CFE has **cost premium of 59%** over 100% RES
- Last 2% more than **doubles cost**

NOTE: CFE = carbon free electricity

Brown, Tom and Iegor Riepin, [Modeling 24/7 Carbon-Free Procurement in Europe](), TU Berlin presentation, June 15, 2022

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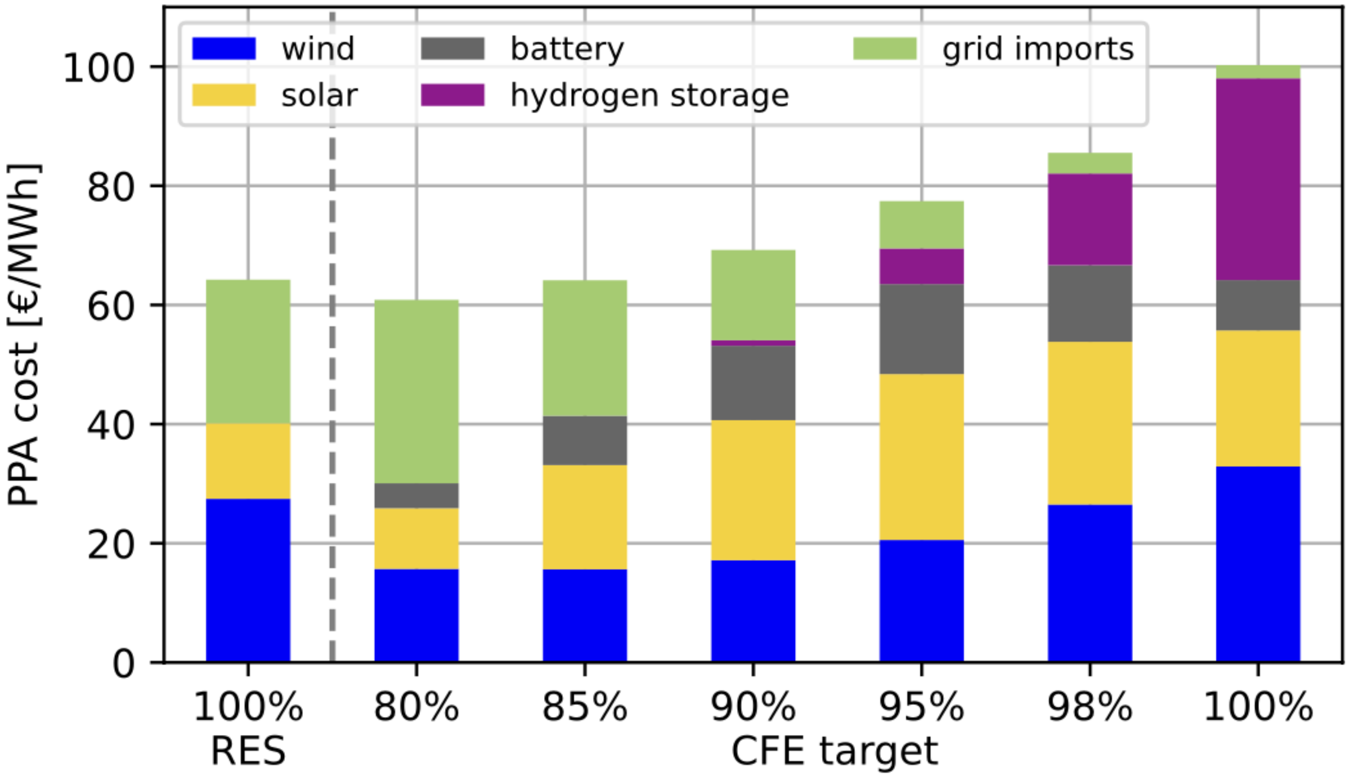
DATA CENTER CASE STUDY – RESULTS WITH LONG DURATION ENERGY STORAGE



- LDES with storage investment cost < 10 €/kWh **limits PPA cost increase**
- With 2 €/kWh hydrogen storage in caverns, cost premium of 100% CFE is **only 56%**

Brown, Tom and Iegor Riepin, [Modeling 24/7 Carbon-Free Procurement in Europe](), TU Berlin presentation, June 15, 2022

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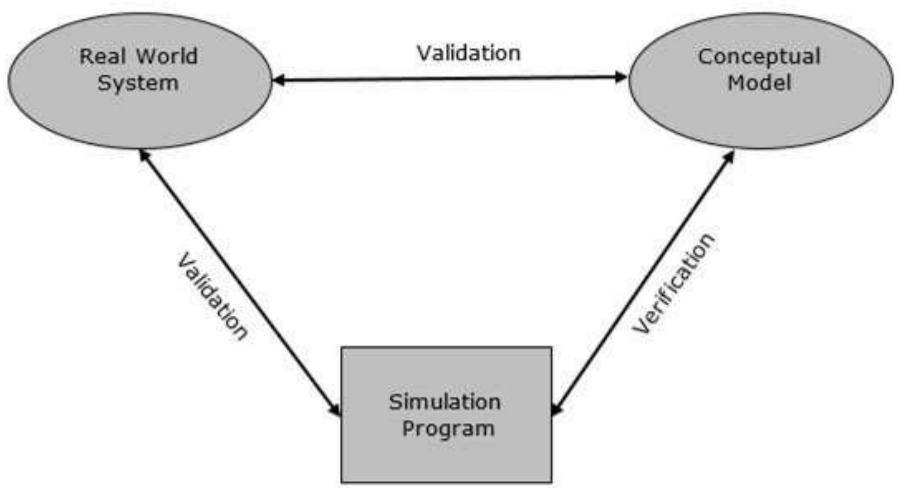
DATA CENTER CASE STUDY – VERIFICATION & VALIDATION

- Verification – making sure the model is doing what is intended
- Validation – making sure the model model matches the phenomenon being modeled



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4. Additional Electric Sector Optimization Applications

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ELECTRIC SECTOR OPTIMIZATION APPLICATIONS

Economic dispatch

Optimal power flow (OPF)

- DCOPF
- ACOPF

Ancillary services

Unit commitment

- Hydroelectric scheduling
- Energy storage

Capacity market auctions

FTR/FTC/TCC auctions

Maintenance scheduling

Generation expansion

Transmission expansion

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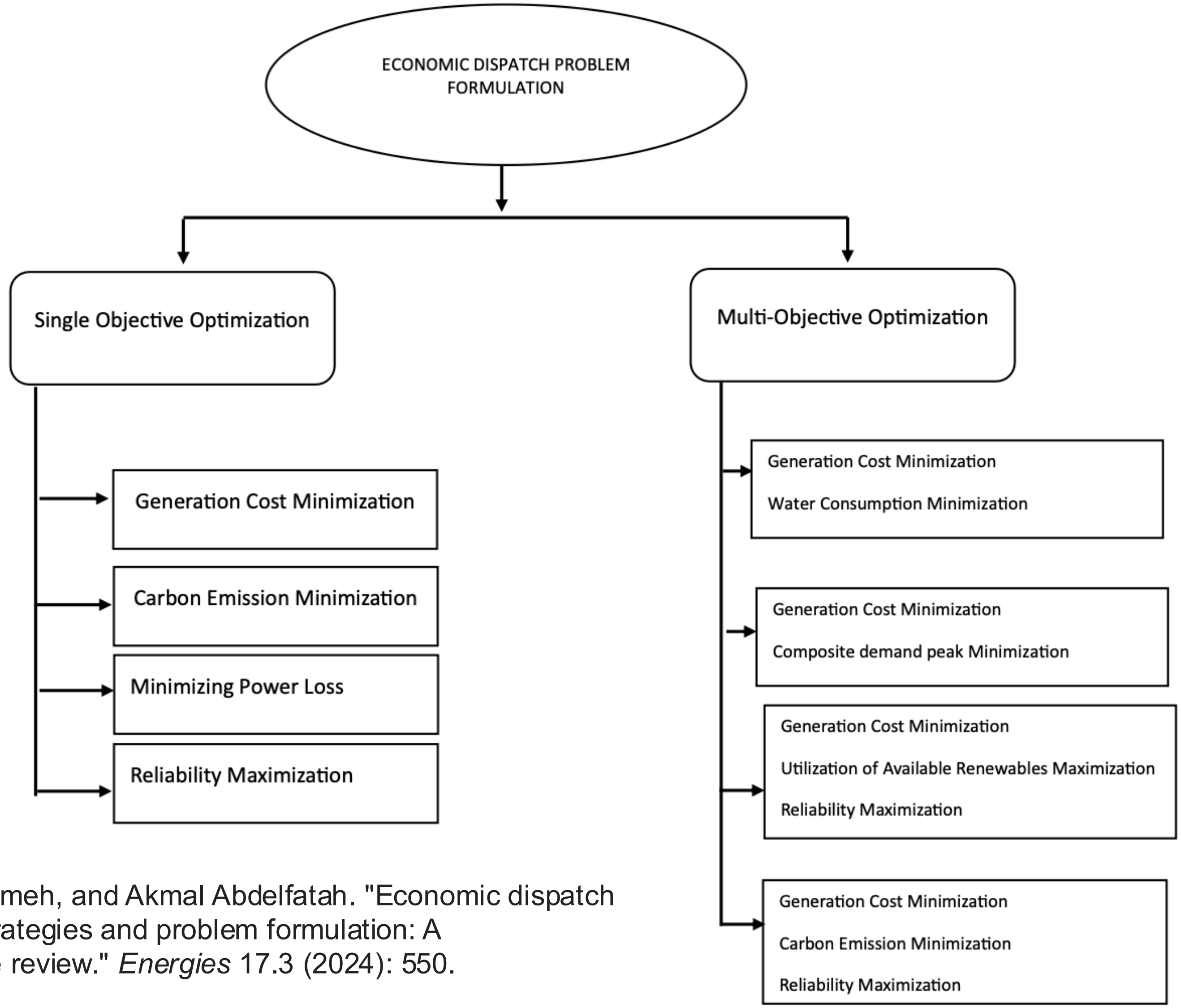
ECONOMIC DISPATCH ALTERNATIVE OBJECTIVE FUNCTIONS



Marzbani, Fatemeh, and Akmal Abdelfatah. "Economic dispatch optimization strategies and problem formulation: A comprehensive review." *Energies* 17.3 (2024): 550.

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OPTIMIZATION OF ENERGY AND RESERVES

Operating reserves are 10-minute spinning, 10-minute non-spinning and 30-minute reserves to help meet generation and transmission contingencies

Sequential optimization involves first optimizing energy, i.e., economic dispatch, then optimizing operating reserves

Co-optimizing optimizes energy and operating reserves simultaneously

Which is more efficient, sequential or co-optimization and why?

What constraints would you add to the economic dispatch problem to co-optimize operating reserves?

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FINANCIAL TRANSMISSION RIGHTS

The FTR auction criteria lends itself to a security-constrained optimization problem. The optimization problem can be described as follows.

- Objective function

$$\max\left(\sum_{Bids} BidPrice * BidQuantity - \sum_{Offers} OfferPrice * OfferQuantity \right)$$

where *BidPrice* is FTR bid price in \$/MW; *BidQuantity* is the awarded FTR in MW to be determined; *OfferPrice* is FTR offer price in \$/MW; and *OfferQuantity* is the sold amount of FTR offer in MW to be determined. *Bids* and *Offers* are sets including all bids and offers.

Subject to the following constraints:

- $0 \le BidQuantity \le$ Total bid-in quantity
- $0 \le OfferQuantity \le$ Total offered quantity
- Power flow balance constraints at buses
- Branch flow limit constraints
- Generic constraints, e.g., the total injection at a group of buses must be less than, greater than, or equal to a given limit; the sum of flows over a group of branches must be less than, greater than, or equal to a given limit, or a combination of bus injections and branch flows.
- Contingency constraints in the form of linear combinations of nodal injections.
- Equality constraints for the dispatchable FTR quantities cleared at source and sink.

LMP's are determined by the shadow prices of power flow balance constraints at each bus. The shadow prices are by-products of the LP solution and readily available. In addition, shadow prices corresponding to all other constraints are also available in the FTR auction solution.

Ma, Xingwang, David I. Sun, and Andy Ott. "Implementation of the PJM financial transmission rights auction market system." *IEEE Power Engineering Society Summer Meeting,*. Vol. 3. IEEE, 2002.

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FINANCIAL TRANSMISSION RIGHTS – SIMULTANEOUS FEASIBILITY

For a pre-defined list of contingencies and monitored elements, an AC power flow model (active power only) is used to perform contingency analysis for each optimization-based bid-clearing solution. If any one of the pre-defined contingencies is identified to have caused security problems, a set of constraints is constructed for each or a selected number of transmission network element rating violations through sensitivity analysis and passed to the optimization module for enforcement in the next iteration. The contingency constraints so constructed are in the form of linear functions of nodal injections and can be readily integrated in the linear programming based bid-clearing module. SFT and the optimization module iterate until no new contingency violation is detected.

Ma, Xingwang, David I. Sun, and Andy Ott. "Implementation of the PJM financial transmission rights auction market system." *IEEE Power Engineering Society Summer Meeting,*. Vol. 3. IEEE, 2002.

UNIT COMMITMENT

$$\min \underbrace{\sum_t \sum_i z_{it} F_{it}}_{\text{Fixed Costs}} + \underbrace{\sum_t \sum_i g_{it} C_{it}}_{\text{Production Costs}} + \underbrace{\sum_t \sum_i y_{it} S_{it}}_{\text{Startup Costs}} + \underbrace{\sum_t \sum_i x_{it} H_{it}}_{\text{Shutdown Costs}} \quad (1)$$

subject to

power balance $\sum_i g_{it} = D_t = \sum_i d_{it} \quad \forall t, \quad (2)$

reserve $\sum_i r_{it} \geq SD_t \quad \forall t, \quad (3)$

UNIT COMMITMENT, con't

min generation	$g_{it} \geq z_{it} MIN_i$	$\forall\, i,t,$	(4)
max generation	$g_{it} + r_{it} \leq z_{it} MAX_i$	$\forall\, i,t,$	(5)
max spinning reserve	$r_{it} \leq z_{it} MAXSP_i$	$\forall\, i,t,$	(6)
ramp rate pos limit	$g_{it} \leq g_{it-1} + MxInc_i$	$\forall\, i,t,$	(7)
ramp rate neg limit	$g_{it} \geq g_{it-1} - MxDec_i$	$\forall\, i,t,$	(8)
start if off-then-on	$z_{it} \leq z_{it-1} + y_{it}$	$\forall\, i,t,$	(9)
shut if on-then-off	$z_{it} \geq z_{it-1} - x_{it}$	$\forall\, i,t,$	(10)
normal line flow limit	$\sum_i a_{ki}(g_{it} - d_{it}) \leq MxFlow_k$	$\forall\, k,t,$	(11)
security line flow limits	$\sum_i a_{ki}^{(j)}(g_{it} - d_{it}) \leq MxFlow_k^{(j)}$	$\forall\, k,j,t,$	(12)

where the decision variables are:

- g_{it} is the MW produced by generator i in period t,
- r_{it} is the MW of spinning reserves from generator i in period t,
- z_{it} is 1 if generator i is dispatched during t, 0 otherwise,
- y_{it} is 1 if generator i starts at beginning of period t, 0 otherwise,
- x_{it} is 1 if generator i shuts at beginning of period t, 0 otherwise,

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https://home.engineering.iastate.edu/~jdm/ee553/UC.pdf

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UNIT COMMITMENT, con't

Other parameters are

- D_t is the total demand in period t,
- SD_t is the spinning reserve required in period t,
- F_{it} is fixed cost ($/period) of operating generator i in period t,
- C_{it} is prod. cost ($/MW/period) of operating gen i in period t;
- S_{it} is startup cost ($) of starting gen i in period t.
- H_{it} is shutdown cost ($) of shutting gen i in period t.
- $MxInc_i$ is max ramprate (MW/period) for increasing gen i output
- $MxDec_i$ is max ramprate (MW/period) for decreasing gen i output
- a_{ij} is linearized coefficient relating bus i injection to line k flow
- $MxFlow_k$ is the maximum MW flow on line k
- $a_{ki}^{(j)}$ is linearized coefficient relating bus i injection to line k flow under contingency j,
- $MxFlow_k^{(j)}$ is the maximum MW flow on line k under contingency j

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https://home.engineering.iastate.edu/~jdm/ee553/UC.pdf

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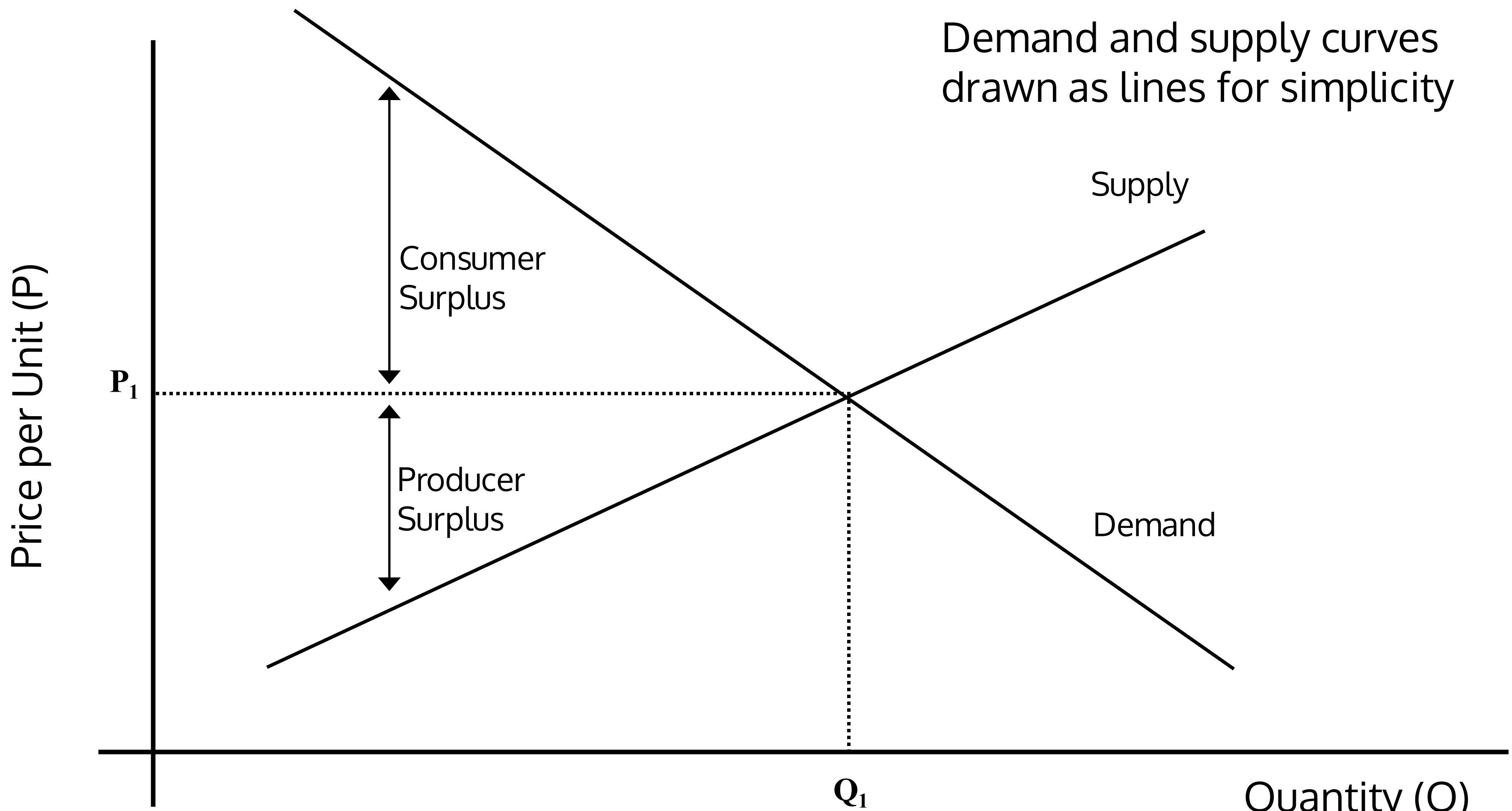
SOCIAL WELFARE MAXIMIZATION



Social Welfare = Consumer Surplus + Producer Surplus

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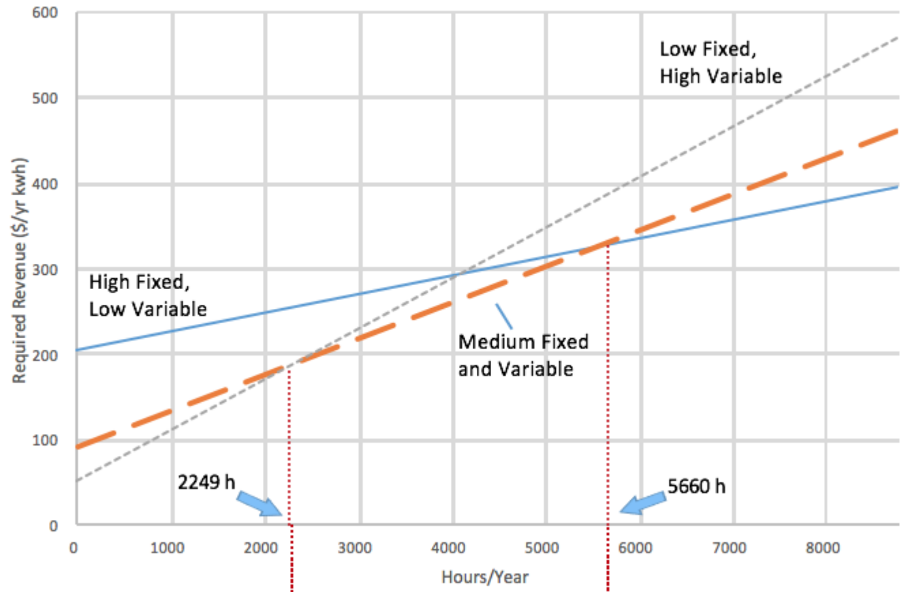
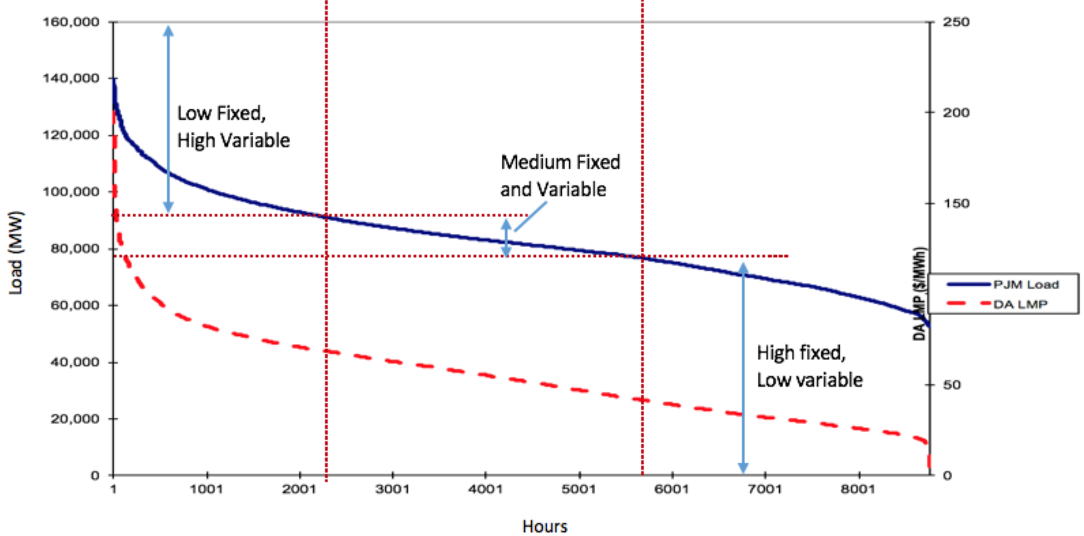
GENERATION INVESTMENT





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COURSE WRAP UP

Optimization is integral to the electric power system

Pricing in all electricity markets explicitly uses an optimization algorithm

Framing challenges as optimization problems is extremely useful

Please contact me at any time if you have questions, comments or want more information

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More Information and Resources

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TERMINOLOGY AND ABBREVIATIONS CHECK

Shadow price
Global minimum
Local maximum
Constraint
Non-negativity constraint
Decision variables
Parameters
First order conditions
Shift factors

LP

x*

SCED

SCUC

MC = MB

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KNOWLEDGE SELF-CHECK

1. What is meant by a feasible solution?
2. What is the economic dispatch problem and what are 3 variations of it?
3. What happens to the optimal value of the solution if the number of decision variables increase? If the number of constraints increase?
4. What are several other applications of optimization in the electric power sector that are not discussed in this presentation?
5. What are the different types of constraints in the unit commitment problem?

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MORE INFORMATION - VIDEOS

Individual Videos

Modeling Electricity Markets with Optimization with Dr. Benjamin F. Hobbs, Nov. 12, 2018

Renewables in Electricity Markets

Optimizing Energy Storage for Ultra-High Renewable Electricity Systems

Video Courses

Math 484 Linear Programming Short Videos, Prof. Wen Shen, 2020

Math 510 Linear Programming and Network Flows, Prof. Henry Adams, 2020

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MORE INFORMATION - DOCUMENTS

Brown, Tom and Iegor Riepin, Modeling 24/7 Carbon-Free Procurement in Europe, TU Berlin presentation, June 15, 2022

Edgeconnex and Gridmatic, Advancing Toward Sustainability: A Year of Operating a Data Center with 24/7 Carbon-free Energy Goals, Sept. 2022

Ma, Xingwang, David I. Sun, and Andy Ott. "Implementation of the PJM financial transmission rights auction market system." *IEEE Power Engineering Society Summer Meeting*,. Vol. 3. IEEE, 2002.

United Nations, The 24/7 Carbon Free Energy Compact

Google, 24/7 by 2030: Realizing a Carbon-free Future, Sept. 2020

Papavasiliou, Anthony. *Optimization models in electricity markets*. Cambridge University Press, 2024.

Wood, Allen J., Bruce F. Wollenberg, and Gerald B. Sheblé. *Power generation, operation, and control*. John Wiley & Sons, 2013.

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