Adding Integers

- Negative + Negative:
  Add the absolute values of the two numbers and make the answer negative.

ex: $-5 + (-9)$
$5 + 9 = 14 \rightarrow \boxed{-14}$

- Negative + Positive (or Positive + Negative):
  Subtract the absolute values of the two numbers (larger minus smaller) and take the sign of the number with the greater absolute value.

ex: $-7 + 12$
$12 - 7 = 5$
12 > 7, so answer is positive
$\rightarrow \boxed{5}$

Subtracting Integers

1. Keep the first number the same
2. Change the subtraction sign to an addition sign
3. Change the sign of the second number
4. Use integer addition rules to solve the new addition problem

ex: $-3 - 9$
$-3 + (-9) = \boxed{-12}$

ex: $-7 - (-4)$
$-7 + 4 = \boxed{-3}$

Multiplying & Dividing Integers

1. Ignore the signs and multiply or divide as usual
2. Determine the sign of the answer using the following rules:
   - positive × or ÷ positive → positive
   - negative × or ÷ negative → positive
   - negative × or ÷ positive → negative
   - positive × or ÷ negative → negative

ex: $-3 \cdot (-5)$
$3 \cdot 5 = 15$
negative · negative → positive → $\boxed{15}$

ex: $48 \div (-6)$
$48 \div 6 = 8$
positive ÷ negative → negative → $\boxed{-8}$

Find the sum or difference.

1. $-80 + 77$	2. $77 + 160$	3. $-64 + (-33)$	4. $104 - (-92)$
5. $-105 - (-122)$	6. $185 - (-154)$	7. $-53 - (-59)$	8. $-6 + (-35)$
9. $15 - (-26) - (-39)$	10. $-93 + 191 + (-179)$	11. $18 + (-34) + 52$	12. $-50 - (-93) + (-17)$

Find the product or quotient.

13. $-60 \div 12$	14. $-194 \div (-2)$	15. $88 \cdot (-2)$	16. $-12 \cdot 10$
17. $-10 \cdot (-11)$	18. $90 \div (-6)$	19. $3 \cdot (-59)$	20. $-7 \cdot (-2)$
21. $-28 \cdot (-6) \div (-24)$	22. $-56 \cdot 14 \div (-8)$	23. $108 \div (-12) \cdot (-12)$	24. $-4 \cdot (-17) \div 2$

Adding & Subtracting Rational Numbers

Determine whether you should add or subtract using integer rules. Then add or subtract.

Decimals:

- Line up the decimal points
- Add or subtract and bring the decimal point down
- Use integer rules to determine the sign of the answer

ex: $-9.8 + 6.24$

neg + pos: subtract

$$\begin{array}{r} 9.80 \\ -\ 6.24 \\ \hline 3.56 \end{array} \rightarrow \boxed{-3.56}$$

ex: $5\frac{3}{4} - \left(-3\frac{7}{8}\right)$

$\rightarrow 5\frac{3}{4} + 3\frac{7}{8}$

pos + pos: add

$$\begin{array}{r} 5\frac{3}{4} = \frac{6}{8} \\ +\ 3\frac{7}{8} = \frac{7}{8} \\ \hline 8\ \ \frac{13}{8} \end{array} \rightarrow \boxed{9\frac{5}{8}}$$

Fractions/Mixed Numbers:

- Find a common denominator and then add or subtract
- Borrow or convert an improper fraction answer, if necessary
- Use integer rules to determine the sign of the answer

Multiplying & Dividing Rational Numbers

Determine the sign of the answer using integer rules. Then multiply or divide.

Multiplying Decimals:

- Ignore the decimal points. Multiply the numbers.
- Count the decimal places in the problem to determine the location of the decimal point in the answer.

ex: $-9.23 \cdot (-1.1)$

neg · neg = pos

$$\begin{array}{r} 9.23 \\ \times\ 1.1 \\ \hline 923 \\ 9230 \\ \hline 10153 \end{array} \begin{array}{l} \leftarrow 2 \text{ dec places} \\ +\ \leftarrow 1 \text{ dec place} \\ \hline 3 \text{ dec places} \\ \downarrow \end{array} \rightarrow \boxed{10.153}$$

Dividing Decimals:

- Move the decimal in the divisor to the end of the number
- Move the decimal in the dividend the same number of places and then bring it straight up in quotient

ex: $-5.2 \div 0.2$

neg ÷ pos = neg

$$02.\overline{)\ 52.} \rightarrow 26. \rightarrow \boxed{-26}$$

Multiplying Fractions:

- Convert mixed numbers to improper fractions.
- Cross-simplify if possible
- Multiply the numerators and multiply the denominators
- Simplify if necessary

ex: $-1\frac{3}{4} \cdot \frac{6}{14}$

neg · pos = neg

$$\rightarrow \frac{\cancel{7}^{1}}{\cancel{4}_{2}} \cdot \frac{\cancel{6}^{3}}{\cancel{14}_{2}} = \frac{3}{4} \rightarrow \boxed{-\frac{3}{4}}$$

Dividing Fractions:

- Convert mixed numbers to improper fractions
- Flip the second fraction to its reciprocal and multiply the two fractions
- Simplify if necessary

ex: $-\frac{1}{2} \div \left(-\frac{3}{8}\right)$

neg ÷ neg = pos

$$\rightarrow \frac{1}{\cancel{2}_{1}} \cdot \frac{\cancel{8}^{4}}{3} = \frac{4}{3} \rightarrow \boxed{1\frac{1}{3}}$$

Find the sum, difference, product, or quotient.

25. $38.61 + 36.841$	26. $1.755 - 1.23$	27. $0.71 \cdot 9.2$	28. $13.12 \div 0.1$
29. $3.651 - (-12.63)$	30. $-3.9 + (-7.6)$	31. $-14.846 \div 2.6$	32. $6 \cdot (-16.7)$
33. $26.474 - 14.527$	34. $-2.1 + 3.78$	35. $-6.15 \div (-8.2)$	36. $-12.8 \cdot (-4.88)$

Find the sum, difference, product, or quotient. Write your answer in simplest form.

37. $15\frac{1}{2} + 15\frac{1}{4}$	38. $18\frac{11}{20} - 17\frac{1}{2}$	39. $3\frac{3}{7} \div 5\frac{1}{3}$	40. $4\frac{1}{2} \cdot 2\frac{2}{5}$
41. $3\frac{1}{3} - 5\frac{1}{9}$	42. $5 \cdot \left(-1\frac{2}{5}\right)$	43. $-7\frac{3}{5} + \left(-3\frac{5}{6}\right)$	44. $-2\frac{1}{12} \div \frac{3}{8}$
45. $9 \div \left(-4\frac{1}{2}\right)$	46. $-18 + 3\frac{4}{5}$	47. $2\frac{5}{6} \cdot \left(-2\frac{2}{3}\right)$	48. $-4\frac{7}{10} - 3\frac{2}{5}$

Order of Operations

Evaluate numerical expressions that contain multiple operations in the following order:

1. Grouping Symbols (complete operations in parentheses, brackets, etc.)
2. Exponents
3. Multiplication & Division (left to right)
4. Addition & Subtraction (left to right)

ex: $-2(-5 + 9)^2 - (-8) + 9$

$-2(4)^2 - (-8) + 9$

$-2(16) - (-8) + 9$

$-32 - (-8) + 9$

$-24 + 9$

→ $\boxed{-15}$

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations

ex: evaluate

$a - bc + b^2$

for $a = -7$, $b = 5$, $c = -1.5$

$-7 - (5)(-1.5) + 5^2$

$-7 - (5)(-1.5) + 25$

$-7 - (-7.5) + 25$

$0.5 + 25$

→ $\boxed{25.5}$

Evaluate the numerical expression. Be sure to use the order of operations!

49. $78 + (-2) \cdot (-56)$	50. $-65 + \frac{6}{-3} + 40$	51. $-94 - [2 - 3(24 - 12)]$	52. $43 + (-23) - (-57)$
53. $-15 - (-11) + 5 \cdot (-4)^3$	54. $-26 - (-64) + (-3)^4$	55. $-84 \div 4 + (-20)$	56. $-56 + (-50) + (-7) \cdot (-9)$
57. $-7.6 - 3 + 2.1 \cdot (-8)$	58. $-\frac{2}{3} + \frac{5}{6} \div \frac{1}{2}$	59. $-8 + 3(-2.7 + 4.23)$	60. $-3\frac{1}{2} \cdot \left(-2\frac{3}{4}\right) + \left(-4\frac{1}{4}\right)$

Evaluate the algebraic expression for $a = -12$, $b = 6$, $c = -4$, and $d = 3$.

61. $a - b + c$	62. $b - cd$	63. $b(cd - a)$
64. $\frac{b}{c} - d$	65. $bd + ac$	66. $\frac{a}{d} + c^2$

One-Step Equations

- Addition Equations:
  Subtract the number being added to the variable from both sides of the equation

ex: $y + 23 = -9$
$-23 \quad -23$
$y = -32$

- Subtraction Equations:
  Add the number being subtracted from the variable to both sides of the equation

ex: $w - 13 = -5$
$+13 \quad +13$
$w = 8$

- Multiplication Equations:
  Divide both sides of the equation by the number next to the variable

ex: $\frac{6x}{6} = \frac{-18}{6}$
$x = -3$

- Division Equations:
  Multiply both sides of the equation by the number under the variable

ex: $\frac{h}{3} \cdot 3 = 4 \cdot 3$
$h = 12$

Two-Step Equations

- Undo operations one at a time with inverse operations, using the order of operations in reverse
  (i.e. undo addition/subtraction before multiplication/division)

ex: $7x - 4 = -32$
$+4 \quad +4$
$\frac{7x}{7} = \frac{-28}{7}$
$x = -4$

- Be sure to always do the same thing to both sides of the equation!

ex: $\frac{j}{5} + 13 = 15$
$-13 \quad -13$
$\frac{j}{5} \cdot 5 = 2 \cdot 5$
$j = 10$

ex: $\frac{b+7}{3} \cdot 3 = -2 \cdot 3$
$b + 7 = -6$
$-7 \quad -7$
$b = -13$

Solve the one-step equation.

67. $19 + j = -34$	68. $m - 26 = 13$	69. $\frac{x}{5} = -3$	70. $12f = 216$
71. $g - (-31) = -7$	72. $\frac{h}{9} = 13$	73. $b + (-3) = -9$	74. $-4w = -280$

Solve the two-step equation.

75. $5m - 3 = 27$	76. $7 + \frac{y}{2} = -3$	77. $4 + 3r = -8$	78. $\frac{1}{2}p - 4 = 7$
79. $\frac{k + 8}{3} = -2$	80. $\frac{f}{5} - (-13) = 12$	81. $-15 - \frac{g}{3} = -5$	82. $-8 + 4m = 2$
83. $-18 - \frac{3}{4}v = 3$	84. $\frac{-5 + n}{4} = -1$	85. $3.5m + 0.75 = -6.25$	86. $2y + 3 = 19$

Unit Rates

- Convert a rate to a unit rate by dividing the numerator by the denominator
  - Write your answer as a fraction with labels for the both the numerator and denominator OR as one number labeled with the first unit "per" the second unit

ex: Find the unit price:

$$\frac{\$2.99}{12 \text{ eggs}}$$

$2.99 \div 12 \approx 0.25$

→ $\boxed{\frac{\$0.25}{1 \text{ egg}} = \$0.25 \text{ per egg}}$

Solving Proportions

1. Set cross-products equal to each other
2. Solve the equation for the given variable

ex: $\frac{5}{b} = \frac{4}{10}$

$5 \cdot 10 = 4b$

$\frac{50}{4} = \frac{4b}{4}$ → $\boxed{b = 12.5}$

Proportion Word Problems

1. Set up a ratio with what you know
2. Set up a second ratio using a variable for the unknown quantity
   - Be sure that the units in the numerator match the units in the numerator of the first ratio and the units in the denominator match the units in the denominator of the first ratio.
3. Make a proportion by setting the two ratios equal to each other
4. Solve the proportion.

ex: A recipe calls for 2 cups of sugar for 36 cookies. How many cups of sugar are needed to make 48 cookies?

$$\frac{2 \text{ cups}}{36 \text{ cookies}} = \frac{x \text{ cups}}{48 \text{ cookies}}$$

$2 \cdot 48 = 36x$

$\frac{96}{36} = \frac{36x}{36}$

$\boxed{x = 2\frac{2}{3} \text{ cups}}$

Convert to a unit rate.

87. $\frac{513 \text{ miles}}{9 \text{ hours}}$	88. $\frac{180 \text{ words}}{5 \text{ minutes}}$	89. $\frac{\$2.53}{8 \text{ oz}}$

Solve the proportion.

90. $\frac{h}{6} = \frac{20}{24}$	91. $\frac{5}{7} = \frac{c}{14}$	92. $\frac{6}{8} = \frac{21}{b}$	93. $\frac{30}{j} = \frac{26}{39}$
94. $\frac{5}{k} = \frac{15}{20}$	95. $\frac{32}{112} = \frac{a}{14}$	96. $\frac{16}{7} = \frac{18}{g}$	97. $\frac{w}{60} = \frac{15}{200}$

Use a proportion to solve the word problem.

98. A cookie recipe calls for 2 eggs and 3 cups of flour. You only have 1 egg, so you have to cut the recipe. How much flour should you use?	99. Jack can run 2 miles in 15 minutes. At that rate, how far would you expect him to run in an hour?	100. Sue read 15 pages of her book in 25 minutes. At that rate, how long will it take her to read the next 10 pages?	101. The ratio of cats to dogs at the park was 1:4. If there were 12 dogs, how many cats were at the park?
102. If 2 pounds of apples cost \$2.60, how much would 5 pounds of apples cost?	103. If you burn 184 calories running 2 miles, how many calories would you burn if you run 5 miles?	104. In a shipment of 300 parts, there are 12 defective parts. How many defective parts would you expect to find in a shipment of 1,000 parts?	105. The ratio of 12-year-olds to 13-year-olds in Mr. Wu's class is 5:3. If there are 24 students in the class, how many students are 13 years old?