THE SPECTRUM OF FLUID SHEAR IN A MIXING VESSEL

by

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Abstract

A mixing vessel has a complete spectrum of fluid shear rates and associated fluid shear element sizes. Fluid shear rates associated with average velocity values at a point are related to mixing phenomena associated with larger particles, bubbles and droplets, while shear rates associated with turbulent velocity fluctuations are usually related to phenomena on a molecular—or micron-size scale. Definitions pertaining to these two different shear rates are presented and some general concepts involved in determining their effect are included.

Introduction

A mixing tank contains a complete variety of sizes, shapes and frequencies of shear stresses. The maximum size of the shear stress elements is determined by the largest dimension of the tank and the impeller, and the mechanical energy is eventually transferred to heat through the mechanism of viscous shear.

The term "fluid shear" has been used so indiscriminately over the years that it is desirable to define some of its important parameters that are of practical use today, and to present some interpretations of existing data as well as some new experimental measurements.

It is well to restate the basic relationship which states that all of the power, P, applied to the mixing system produces a circulating capacity, Q, and a velocity head, H, within the discharge, such that

$$P \propto Q\,H \qquad \ldots (1)$$

The use of a large-diameter, slow-speed impeller produces more flow and less head than a small-diameter, high-speed impeller at the same power consumption. Accordingly, processes that depend primarily on the circulating capacity of the mixer have been observed to require less horsepower with large, slow-speed impellers than with small, high-speed impellers.

It must be remembered that in these flow-sensitive kinds of processes there is some minimum level of fluid shear stress or a minimum actual velocity that has been satisfied by all the various impeller diameters used.

Many important mixing processes depend on the level of fluid shear stress, although there must always be a circulation flow into and out of the fluid shear stress zones.

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Fluid Shear Definitions

If the average velocity of the fluid leaving the blades of a radial flow turbine is measured at various positions above and below the impeller centerline, the distribution of radial velocities is typically as shown in Figure 1. The velocity

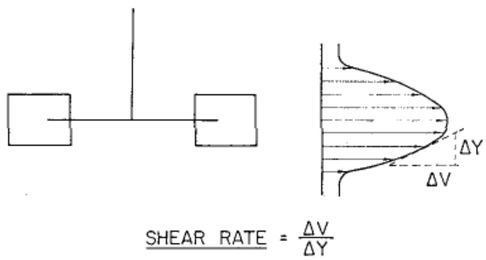

Figure 1. Typical velocity profile from radial flow impellers.

gradient at a point or across some chosen distance, is defined as the fluid shear rate, either at the point or across the chosen distance increment. The size of the liquid droplet, gaseous bubble or solid particle of interest in the particular process determines the distance increment used in making the fluid shear analysis.

It is important to point out that it is the fluid shear stress which ultimately produces the process result. Fluid shear stress is defined as the product of viscosity and fluid shear rate.

$$\text{Fluid Shear Stress} = \mu(\text{Fluid Shear Rate}) \qquad \ldots (2)$$

An article by Karam [5] gives some excellent data to illustrate the difference between shear stress and shear rate. Table I is extracted from cross plots of their

Table I

Shear Rate to Rupture A Given Droplet Size

	$\mu = 80$ Cp	
μ Cp	Shear rate sec.$^{-1}$	Shear stress in contin. phase gm./cm.^{2}
80	93	0·08
240	38	0·09
370	22	0·08
745	16	0·12
1175	14	0·17

μ = continuous phase viscosity
μ' = discontinuous phase viscosity

data showing the shear rate required with different continuous phase viscosities at one dispersed phase viscosity to break up the same size droplet. They show that the shear stress in gm./cm.^{2} is the basic parameter and that viscosity and shear rate are inversely proportional to give the required shear stress.

There are four regions in a tank which have become of particular interest when studying mixing operations. The fluid shear rates are defined first for average velocity measurements at a point. These definitions hold true for both laminar and turbulent flow.

1. The average velocity gradient measured throughout the impeller zone.
2. The maximum velocity gradient measured in the impeller zone.
3. The average velocity gradient measured throughout the entire tank volume.
4. The minimum velocity gradient in the tank volume.

If the fluid flow in the tank is turbulent, then a typical velocity pattern at a point is illustrated by Figure 2. The previous definitions are based on the average velocity, $\bar{u}$ [7].

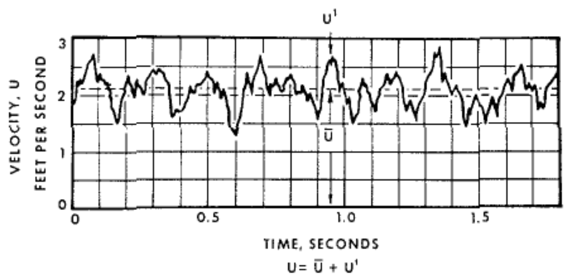

Figure 2. Typical velocity versus time recorded for turbulent flow at a point. Schematic only, not from this reported research.

We may also examine the turbulent velocity fluctuations, u', and make measurements of some of the pertinent quantities.

In general, these turbulent velocity fluctuations are of much higher frequency and much smaller size, and operate over much smaller dimensions than do the fluid shear rates based on average velocities.

Turbulent fluctuations will be considered in a later section.

Data presented by Oldshue [7] confirm that the average impeller-zone shear rate is a function only of impeller speed, as presented by Metzner [6], while the maximum impeller-zone shear rate is primarily a function, for a given geometric series of an impeller type, of the peripheral speed of the impeller. Figure 3 illustrates this point. As an example, the average shear rate on Figure 3 is about 30 sec.$^{-1}$ at 100 RPM.

On scaling up to larger size systems, there is a greater difference between maximum impeller zone shear rates and average impeller zone shear rates (fig. 10).

The average velocity gradient throughout the entire tank can be defined by several different methods. Estimates from this laboratory indicate that these velocity gradients are an order of magnitude less than the average velocity gradient around the impeller zone.

The minimum velocity gradient in the tank has been estimated as one-quarter to one-third of the average velocity gradient throughout the entire tank.

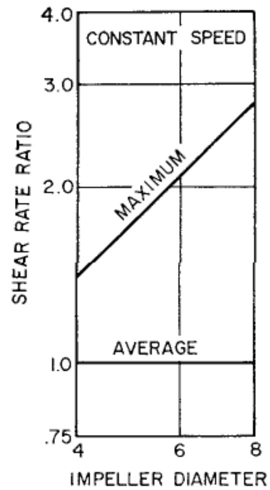

Figure 3. Relationship between maximum impelled-zone shear rate and average impeller zone shear rate for turbine diameter at constant speed.

Turbulent Velocity Fluctuations

The experimental data reported by Oldshue [7] were obtained with a hot-wire velocity meter. The details of that equipment are included in an Appendix to this report. Additional data from this particular equipment are shown in Figures 4, 5 and 6. Figure 4 shows some of the data which give the ratio of the RMS velocity fluctuation $\sqrt{u'^2}$ to the average velocity at that point as a function of position above and below the impeller centerline. These data are for a 4-in. diameter 6-flat blade turbine at 259 RPM. This indicates the ratio of fluctuating velocity to average velocity at a point which can be as high as $0 \cdot 75$, and is generally in the $0 \cdot 5$ to $0 \cdot 7$ range. These measurements were made approximately three-quarters of an inch from the impeller periphery. Additional measurements made in other

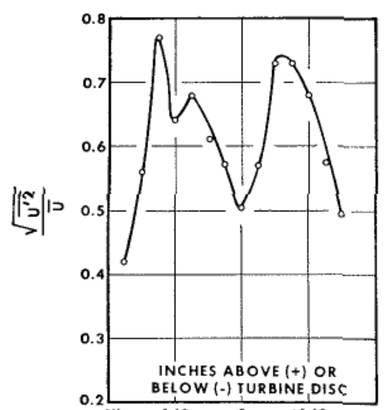

Figure 4. Ratio of root mean square velocity fluctuations to mean velocity at a point and distance above impeller centreline.

positions in the tank are shown in Figure 5. These indicate that the velocity fluctuations in other parts of the tank are in the range of 0·05 to 0·15 of the average velocity which is more typical of turbulence behind grids and in pipelines.

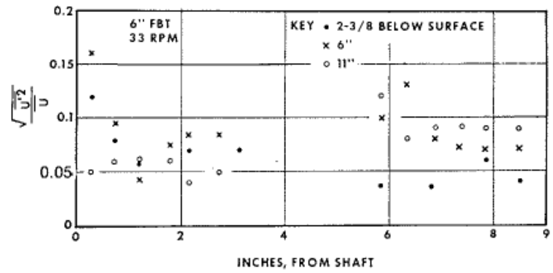

Figure 5. Ratio for root mean square velocity fluctuation and average velocity at a point for various positions in a mixing tank away from the impeller zone.

Other background on turbulent velocity parameters is given by Cutter [2] who calculated several parameters from photographs of flow patterns in a mixing vessel. He found that the RMS velocity fluctuation was primarily related to the energy dissipation. His measurement scale in making these photographic studies could be quite different from the scale of velocities from the hot-wire velocity meter reported here, so that a direct comparison of these results with those of Cutter has not yet been made.

It appears that the frequency and the size of the turbulent velocity fluctuations are more compatible with phenomena on the molecular- or micron-size scale in a mixing vessel. For example, in a recent presentation, Paul and Treybal [9] show that the yield of a given reaction which had several alternate paths was determined by the RMS velocity fluctuation at the feed point. Paul used data from Schwartzberg and Treybal [10] and Cutter to calculate RMS velocities at the feed point.

It appears now that the turbulent RMS velocity fluctuation is more related to processes occurring on a scale in which the active particles are 200-mesh size or

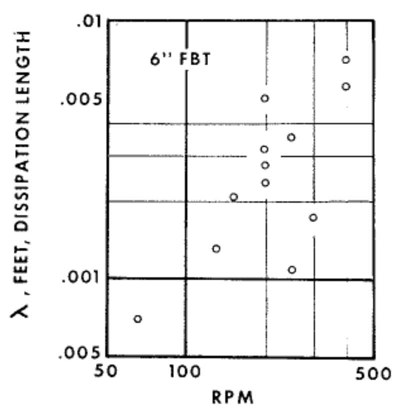

Figure 6. Turbulent dissipation length in the impeller zone as a function of impeller speed.

smaller. There are undoubtedly processes where the fluid shear rates based on average velocities at a point, as well as turbulent shear rates based on the RMS fluctuation values, are important to the particular process. It is important to design experiments which can separate these two effects.

Another parameter which can be measured by the hot-wire velocity meter technique is the dissipation length, λ. This is helpful in indicating the size of the turbulent eddies. λ is a measure of, although not equal to, the mean size of the smaller eddies which are dissipating turbulent energy. Figure 6 gives some experimental values. At the moment, λ values are not being used to quantitatively analyse processes in a mixing vessel.

Some Practical Considerations

Several examples of shear-sensitive processes will illustrate the importance of these concepts. The effect of fluid shear stress on the biological solids in fermentation and waste treatment, the effect of fluid shear stress on particle size and coagulation in emulsion and suspension polymerization, and the effect of fluid shear stress on pigment dispersion in coatings and fibers are examples.

There are two very different types of dispersion processes. The first type consists of friable or fragile particles whose dispersion is irreversible.

In this situation once a particle is affected by a shear stress and some particular particle size is produced, there is no tendency for recombination or recoalescence in the rest of the tank. The maximum impeller-zone shear rate is an indication of the minimum size particle that can be produced with sufficient time and circulating capacity through the shear zone.

There is usually a threshold shear stress below which nothing can happen as well as an inverse relationship between particle size and time in arriving at this point.

The second type of dispersion process occurs with reversible and dynamic system in which droplets produced in the impeller zone can coalesce and recombine in the lower shear stress zones in the rest of the tank.

The particle size existing in various parts of the tank at a given time is a function of average impeller zone shear rates, maximum impeller zone shear rates, average tank zone shear rates, and minimum tank zone shear rates, and the kinetics of dispersion and coalescence for the particular system.

Howarth [3], indicates an interesting technique for estimating coalescence frequency by quickly lowering impeller speed from a particular steady-state condition.

Since actual systems may involve dispersion phenomena in between these two extremes, it can be seen why different experimenters have shown different kinds of relationships between mixer variables depending upon which type of shear stress is important in that particular process.

Average Shear Rates and Shear Stresses

Metzner [6] measured the power response of an impeller with both pseudo-plastic and Newtonian fluids and established constants for the shear rate with a rotating impeller. Coyle, *et al.* [1] added some data for close-clearance helical impellers. These measurements are made in the laminar region of the Reynolds number - Power number curve shown by Figure 7. By measuring the power, speed and diameter, the viscosity a pseudoplastic fluid exhibits can be picked off of Figure 7. Then, from a curve of viscosity versus shear rate for the pseudoplastic fluid, Figure 8, the shear rate around the impeller can be estimated.

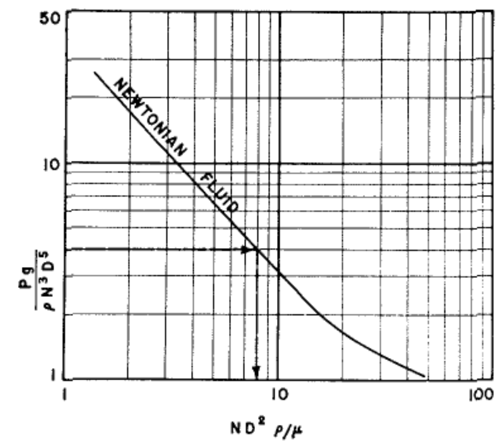

Figure 7. Typical Reynolds number—Power number curve for Newtonian fluid.

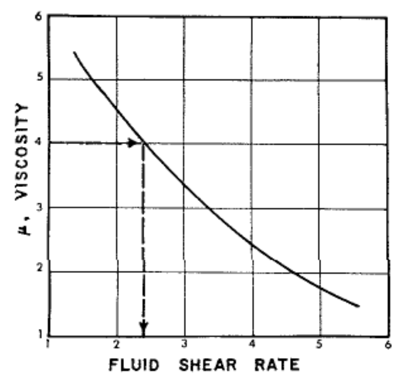

Figure 8. Typical viscosity versus shear rate for pseudoplastic fluid.

This particular calculation means that the velocity distribution around the impeller determines the average shear rate that has been measured. It appears that a wide variety of different pseudoplastic fluids in the laminar region give the same calculated shear rate by this method.

However, in making these kinds of measurements in the transition region of the Reynolds number - Power number curve, it is found that the measured shear rates are different from those for the laminar region. The accuracy of the measurement is less because of the lesser effect of viscosity in this region. There apparently is a difference in the velocity distribution produced by the impeller in this region as compared with that for the laminar flow region.

Average Tank Volume Shear Rate

It is possible to define the average tank shear rate by using a mixing process with a pseudoplastic fluid. As an illustration, consider a blending process in which the power required to obtain a constant blend time using an arbitrary standardized blending technique has yielded a curve with a series of Newtonian fluids as shown on Figure 9. By using a pseudoplastic fluid whose shear rate versus shear stress

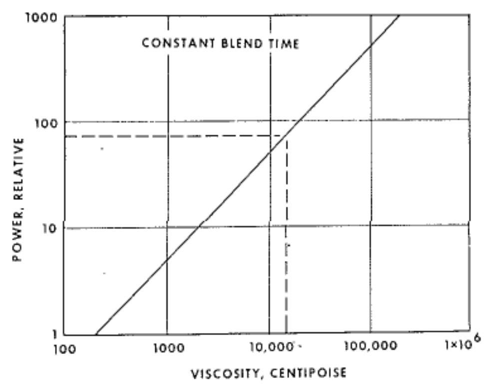

Figure 9. Typical power consumption versus viscosity for a constant blend time in Newtonian fluid.

relationship is known, Figure 8, and measuring blend time, and the power required to achieve the same blend time, the viscosity of the pseudoplastic fluid equivalent to that of a Newtonian fluid can be determined, Figure 9. The average tank shear rate can then be estimated from Figure 8. Data obtained by these techniques indicate to date that the average tank shear rate is an order of magnitude below the average impeller zone shear rate.

A Scale-up Example

Referring to Figure 10 it is seen that the maximum impeller-zone shear rate tends to increase on scale-up, while the average impeller-zone shear rate tends to

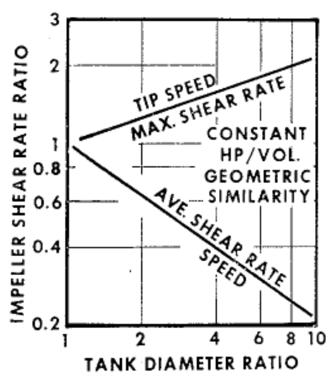

Figure 10. Change in maximum impeller-zone shear rate and average impeller-zone shear rate with scale-up.

decrease [8]. If it is desired to keep the maximum impeller-zone shear rate below some maximum value, then there usually must be a change in the impeller-to-tank diameter ratio or blade geometry. As one example of this process, refer to Figure 11 which shows the yield of product per pound of catalyst on a relative basis at different power levels for a particular geometry in a 10-gallon pilot plant.

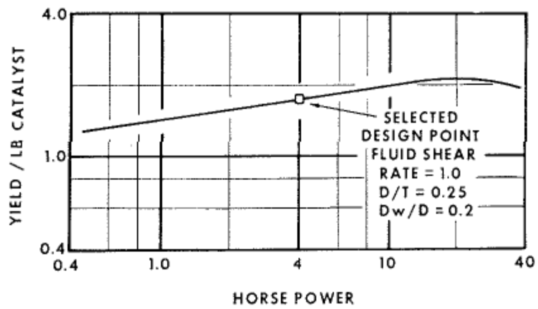

Figure 11. Product yield versus horsepower for a typical pilot-plant experiment.

Scaling up to a 3400-gallon tank, a mixer with the same geometry was designed to achieve the same gas-liquid mass transfer rate and the same liquid-solid mass transfer rate, Table II, Column B. The maximum impeller-zone shear

Table II

Fluid Mixing Properties on Scale-up

	Pilot plant A	B	C	D
T, ft.	1·2	7·0	7·0	7·0
P, Relative	1·0	340	340	340
N, Relative	1·0	0·31	0·16	0·075
D/T	0·25	0·25	0·38	0·6
No. of impellers	1	2	2	2
Z, ft.	1·2	12·0	12·0	12·0
F, ft./sec.	F	10F	10F	10F
Liq.-solid mass transf. rate/vol.	1·0	1·0	1·0	1·0
Gas-liq. mass transf. rate/vol.	1·0	$>>$ 1·0	$>>$ 1·0	$>>$ 1·0
Max. impeller-zone shear rate	1·0	1·8	1·4	1·0
Torque	—	1·0	2·0	4·1

rate in the full scale unit is approximately 80% higher than it was in the pilot plant. If this selection B were installed, a serious process deficiency could have resulted. Referring to Figure 12, pilot-plant runs with a different impeller geometry, which gave maximum impeller-zone shear rates more typical of those in the plant at the same mass transfer conditions, indicated that the catalyst would be degraded by these shear rates and that process performance would suffer.

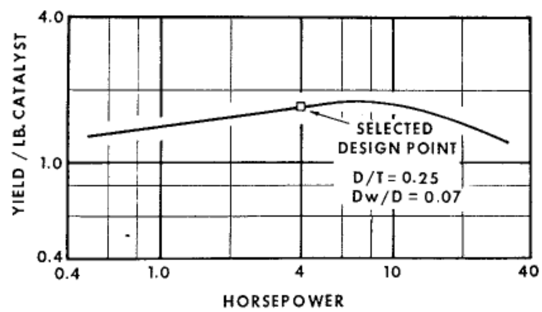

Figure 12. Product yield versus horsepower for a narrow-blade turbine compared to Figure 11.

A second mixer, Column C, for the full scale plant to achieve the desired gas-liquid and liquid-solid mass transfer rates with a 40% increase in maximum impeller-zone shear rate, was designed. It should be noted here that the second mixer, Column C, required to lower the shear rates, is more expensive than the original selection, Column B, since it must operate at a lower speed and therefore requires a higher torque mixer drive.

Column D shows what would be required to lower the maximum impeller-zone shear rate to the same level as the initial pilot plant, Figure 11.

Appendix

Hot-Wire Velocity Probe Technique

To obtain more information on the nature of flow from a turbine impeller, a hot-wire velocity meter was developed for use in mixing systems. The measuring element consists of a small wire suspended between two electrodes. The wire was 0·100 in. long and 0·0007 in. in diameter. The wire was heated electrically to about 20°F above the tank temperature. As the water flows by the probe, it tends to cool it, and thus the amount of current required to maintain constant temperature is related to the velocity of the fluid.

The electrical arrangement of the equipment is as described by Hubbard [4]. A hot-wire probe gives the maximum flow rates when the wire is at right angles to the flow stream. In this study, the wire was always positioned in a horizontal plane and was rotated at various angles to a radius from the centerline of the tank until maximum velocity readings were obtained. From this study it appeared that the angle for maximum velocity corresponded to a tangential velocity component at the impeller periphery.

In the data reported here, the probe was positioned three-quarters of an inch away from the turbine periphery, directed along a tangent to the impeller periphery, and was positioned at various distances above and below the turbine centerline.

The tank diameter, T, used in this work was 18 in., and the liquid level, Z, was 20 in. Impeller diameters, D, used were 4, 6 and 8 in. Four baffles were used in the tank, each $1 \cdot 5$ in. wide. The impeller was placed 6 in. off bottom.

The hot-wire velocity meter has a very fast response, so that the velocity u, at a point at any time can be expressed as a mean velocity over a time interval, $\overline{u}$, plus a fluctuating velocity component, u', so that

$$u = \overline{u} + u' \qquad \ldots (3)$$

The probe was calibrated by placing it into the flow from an orifice in a tank with a constant head for the period of calibration. The orifice was constructed very carefully in accordance with fluid mechanics standards, and the flow through the orifice calculated from a knowledge of the static head and the orifice coefficient.

Symbols Used

D	Impeller diameter.
D/T	Impeller diameter-to-tank diameter ratio.
D_w	Impeller blade width.
$\dfrac{ND^2\rho}{m}$	Reynolds number, ratio of inertia force to viscosity force, dimensionless.
P	Total power.
$\dfrac{Pg}{\rho N^3 D^5}$	Power number, ratio of applied force to gravitation force, dimensionless.
Q	Volumetric fluid displacement of impeller.
RMS	Root mean square, $\sqrt{\overline{u'^2}}$
T	Tank diameter
u	Velocity in x direction.
$\overline{u}$	Mean velocity, ft./sec.
u'	Fluctuating velocity.
Z	Liquid depth.
λ	Dissipation length. $\left[\dfrac{1}{\lambda^2} = \dfrac{1}{\overline{u^2}} \overline{\left(\dfrac{du^2}{dx}\right)}\right]$
μ	Continuous phase viscosity.
μ'	Discontinuous phase viscosity
ρ	Density of fluid or solid.

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