Fluid Mixing, Heat Transfer and Scale-up

By J. Y. Oldshue,* M.S., Ph.D.

Factors involved in describing fluid motion in a mixing vessel include the pumping capacity of the impeller and the fluid shear rate that exists at various points in the vessel. Measurements of fluid shear are presented as well as its use in mixer scale-up. Heat transfer data on several types of impellers in tanks equipped with helical coils and wall baffles are discussed by the author, with a general correlation of heat transfer coefficients *vs.* Reynolds number.

MIXING of fluids is an important unit operation. In discussing mixing, it is necessary to separate out the different aspects in order to draw the proper conclusions.

For purposes of discussion, the process considerations are divided into three areas as shown in Table 1. The first area, (I–1), is a study of the fluid regime produced in a mixing vessel without regard to the effect of this fluid regime on any particular mixing process. This involves consideration of pumping capacity, fluid shear, flow patterns and other concepts of fluid mechanics. In studying the fluid mechanics of flow in fluid mixing systems, we must realise that no conclusion on mixing effectiveness can be drawn until some particular mixing result is desired, in which it is assumed or known that a particular flow pattern is required.

The second area, (I–2), is consideration of the requirements of various processes for mixing results. Since the field of mixing has such a vast range, there are many different criteria for successful mixer performance. Table 2 gives a convenient subdivision of the field of mixing into five basic application classes, which cover the various types of fluids involved. Then there are two subdivisions depending upon the type of process mechanism involved. The five basic application classes in Table 2 are liquid/solid, liquid/gas, immiscible liquids, blending of miscible liquids and fluid motion. The two basic areas in each of these classes are, first of all, for those processes where a physical degree of uniformity is the criterion, and secondly, those areas where some type of mass transfer rate between the phases or a chemical reaction rate within a phase is required.

One example illustrates the usefulness of this classification. There are many similarities between the mixing requirements of all the types of solid suspension operations where the criterion is expressed as the degree of solid suspension in various parts of the tank. As one would expect, these criteria are quite different from those involved in effective gas absorption in a gas/liquid contacting operation.

Table 1. Elements of Mixer Design

I	Process design ..	1	Fluid mechanics of impellers
		2	Fluid regime required by process
		3	Scale-up; hydraulic similarity
II	Impeller power characteristics	4	Relate impeller horsepower, speed and diameter
III	Mechanical design	5	Impellers
		6	Shafts
		7	Drive assembly

The third area under (I) in Table 1, process design, incorporates knowledge of the scale-up parameters involved in mixing (I–3). As would be expected, important scale-up parameters depend upon the type of mixing operations under study.

Another area distinct from process considerations is the power (II–4) drawn by the impeller. For given fluid properties, the power of an impeller is a function of the impeller speed, impeller diameter and, to some degree, the impeller position in the tank.

Data which relate the power consumption of an impeller to these variables do not shed any light at all on whether a given condition will produce a satisfactory mixing result. In some of the discussion following, comparison will be made on the basis of constant power input to a mixing system. This means that as the impeller has been changed, the speed has been adjusted to give equal power input.

The discussion in this article is related first of all to consideration of the flow and fluid shear from mixing impeller, heat transfer in mixing systems and how some of these parameters change in scale-up.

Flow in mixing vessel

The pumping capacity of flat-blade turbines has been measured by various investigators. Fig. 1 shows a typical installation of a flat-blade turbine in a baffled mixing tank. The flat-blade turbine referred to here is of the type shown in Fig. 2. It has six blades and the dimensions are shown on the figure. One of the most complete works on pumping capacity is that of Sachs,[4] who used the photographic technique of analysing velocity patterns. To illustrate the type of flow pattern, Figs. 3 and 4 are included from his

*Dr. J. Y. Oldshue is technical director of the Mixing Equipment Co., which he joined in 1950. He graduated at the Illinois Institute of Technology, and is the author of numerous articles in the field of mixing in such areas as chemical processing, paper manufacture, food processing and petroleum processing.

Table 2. Classification of Mixing Process Results

Application classes	*Mixing results*	
	Chemical processing	*Physical processing*
Liquid/solid	*Dissolving, crystallising:* Sugar dissolving, ore leaching, vacuum crystallisers	*Suspension and dispersion:* Filter aid, clay slurrying, paper pulp
Liquid/gas	*Absorption and stripping:* Aeration, hydrogenation, steam stripping	*Dispersions:* Lightweight concrete, degassing, foam control
Immiscible liquids	*Extraction:* Solvent extraction, acid treating, caustic washing	*Emulsions:* Wax polishes, cosmetics, salad dressing
Miscible liquids	*Reactions:* Waste neutralisation, pH control, polymerisation	*Blending:* Vegetable oils, gasoline, paint
Fluid motion	*Heat transfer:* Coils and jackets, quench tanks, melting and freezing	*Pumping:* Pump-mix extractors, lift pumps, plating circulators

NOMENCLATURE

B = baffle width
Btu = British thermal units
C = impeller distance from tank bottom
C_c = height of coil from bottom of tank
C_s = height of sparge from bottom of tank
C/D = off-bottom distance to impeller diameter ratio
C_p = specific heat of liquid
D = impeller diameter
D_b = turbine blade length
D_c = coil diameter
D_d = disc diameter
D_s = diameter of sparge ring
D_w = blade width (projected vertical height)
d = outside diameter of heat transfer tube
d/T = heat transfer tube to tank diameter ratio
D/T = impeller diameter to tank diameter ratio
ft^3/min = cubic feet per minute of air flow
h = heat transfer coefficient
h_c = cooling coefficient
h_h = heating coefficient
h_o = coefficient calculated from heating and cooling coefficients h_h and h_c given in Equation (3)
k = *thermal conductivity of liquid*
m = exponent as given in paper by Oldshue and Gretton[3]
N = impeller rotational speed, rev/min
$ND\pi$ = peripheral speed
N_{Re} = Reynolds number, ratio of inertia force to viscosity force, $ND^2\rho/\mu$
P = total power
P/V = power per unit volume of liquid
Q = flow from the impeller per unit time
Q/V = flow per unit time per unit volume of tank
rms = root mean square
S_c = spacing between heating coils
T = tank diameter
t = temperature
U = overall heat transfer coefficient
u = velocity
$\bar{u}$ = mean velocity over a time interval
u' = fluctuating velocity component
V = volume
y = height above impeller centreline horizontal plane
Z = liquid depth
Z_c = height of helical coil
ρ = density of fluid or solid
μ = fluid viscosity at temperature of fluid in tank
μ_s = viscosity at surface temperature
Δ_c = log mean concentration, ppm
Δ_t = log mean temperature difference, °F

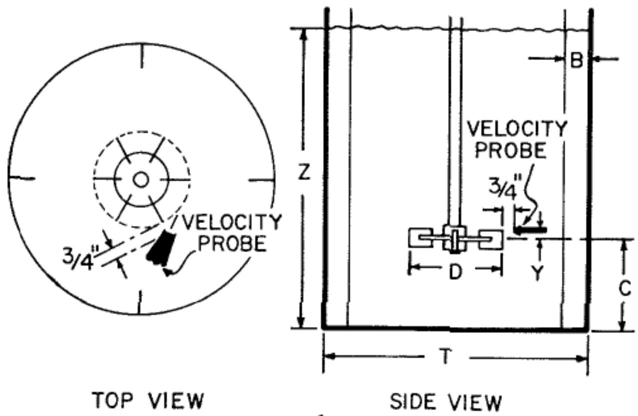
Table 3. Flow Characteristics as Measured with the Hot-wire Velocity Meter

Diameter (in) D	*Impeller speed (rev/min)* N	*Maximum laminar shear rate,* $d\bar{u}/dy$ (s^{-1})	*Mean laminar shear rate,* du/dy (s^{-1})	*Mean rms value* $\sqrt{\bar{u}'^2}$ *(ft/s)*
4	257	88·5	62·4	1·02
6	33	20·1	13·5	0·17
6	66	32·3	19·4	0·42
6	131	78·7	39·3	0·97
6	250	106·2	65·0	1·86
8	100	100·0	29·3	1·01

Table 4. Scale-up of a Mixing Process

Properties	*Pilot scale*	*Plant scale*	
		Design to satisfy mass transfer	*Design to reduce fluid shear rate*
T, tank diameter	12 in	8 ft	8 ft
P, impeller horsepower ..	0·09	57	66
D, impeller diameter ..	4 in	34 in	44 in
Number of turbines ..	1	3	3
Z, liquid level	12 in	20 ft	20 ft
Volume, gal	6·0	7,500	7,500
Air flow, ft^3/min	0·46	114	114
F, ft/s	0·01	0·027	0·027
Total heat, Btu/h.. ..	4,500	5,650,000	5,650,000
Δ_t, °F	40	80	80
Area for heat transfer, ft^2	0·75	565	565
U, Btu/h ft^2 °F	150	125	130
Mass transfer rate, ($K_La \times \Delta_c$), ppm/h ..	1,860	1,860	1,860
K_La, h^{-1}	200	250	250
Δ_c log mean concentration, ppm	9·3	7·5	7·5
Percentage active gas absorption	15	75	75
**Laminar shear rate*			
In turbine jet, max. ..	1·0	1·8	1·45
In turbine jet, av. ..	0·5	0·11	0·066
**Turbulent shear intensity*			
In turbine jet, av. ..	10·0	18·0	14·5
In tank flow, av. ..	1·0	0·51	0·59
Flow from impeller/tank vol.	19·4	6·4	10·4
HP above minimum for complete suspension ..	10·0	2·0	2·5

*These are ratios only. There is no numerical correspondence between laminar shear rates and turbulent shear intensity.



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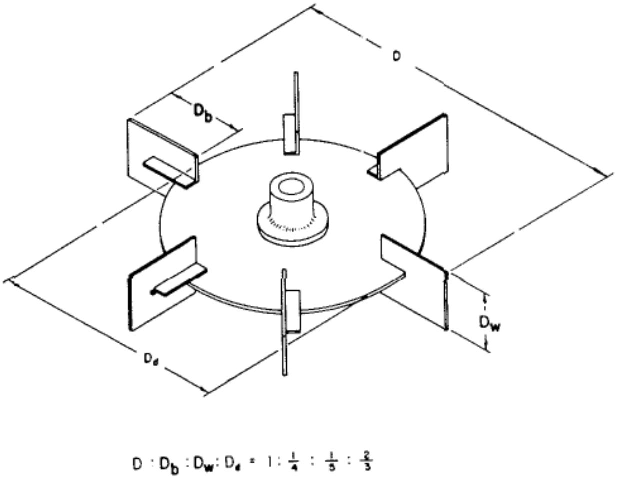

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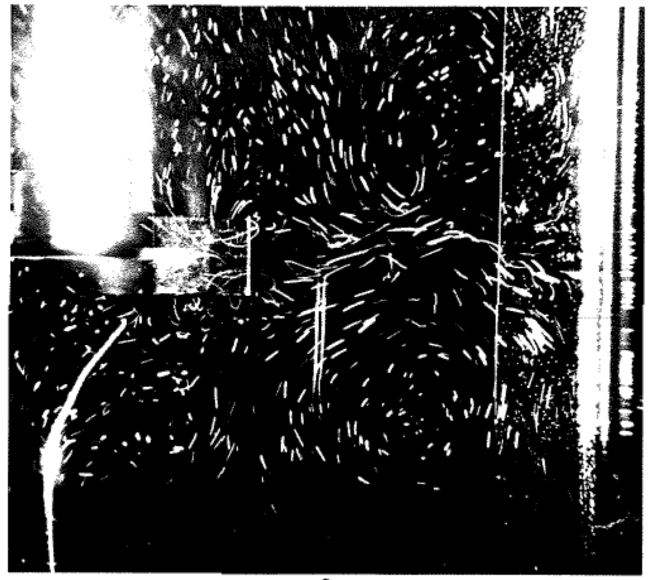

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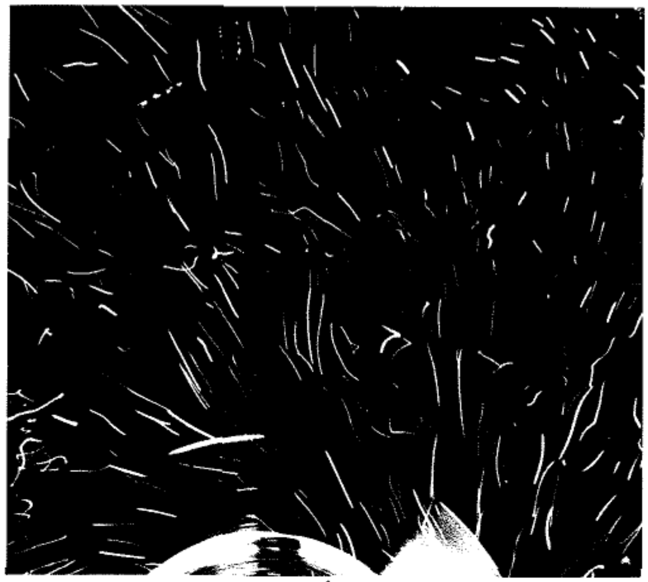

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Fig. 1. Typical turbine installation in baffled tank, showing location of hot-wire velocity probe

Fig. 2. Typical dimensions of flat-blade turbine impeller

Fig. 3. Flow pattern, side view, 4-in-d flat-blade turbine, 12-in-d tank, water; streak photograph showing particle velocities in $\frac{1}{4}$-in wide vertical plane

Fig. 4. Typical flow pattern from flat-blade turbine in a baffled tank, bottom view, 4-in-d turbine, 12-in-d tank, water, $\frac{1}{4}$-in wide horizontal plane at impeller centreline

work, showing the overall type of flow developed by a flat-blade turbine.

To obtain more information about the nature of flow from a turbine impeller, a hot-wire velocity meter was developed for use in mixing systems. The measuring element consists of a small wire suspended between two electrodes. The wire was 0·100 in long and 0·0007 in *d*. The wire was heated electrically to about 20°F above the tank temperature. As the water flows by the probe it tends to cool it and the amount of current required to maintain constant temperature is related to the velocity of the fluid.

The electronic arrangement of the equipment is as described by Hubbard.[1] A hot-wire probe gives the maximum flow rates when the wire is at right-angles to the flow stream. In this study the wire was always positioned in a horizontal plane and was rotated at various angles to a radius from the centreline of the tank until maximum velocity readings were obtained. From this study it appeared that the angle for maximum velocity corresponded to a tangential velocity component at the impeller periphery.

In the data reported here, the probe was positioned $\frac{3}{4}$ in away from the turbine periphery, directed along a tangent to the impeller periphery, and was positioned at various distances above and below the turbine centreline.

The tank diameter, T, used in this work was 18 in, and the liquid level, Z, was 20 in. Impeller diameters, D, used were 4, 6 and 8 in. Four baffles were used in the tank, each $1\frac{1}{2}$ in wide.

The hot-wire velocity meter has a very fast response so that the velocity, u, at a point at any time can be expressed as a mean velocity over a time interval, $\bar{u}$, plus a fluctuating velocity component, u', so that

$$u = \bar{u} + u' \quad \ldots\ldots\ldots\ldots\ldots (1)$$

The probe was calibrated by placing it into the flow from an orifice in the tank with a constant head for the period of the run. The orifice was constructed very carefully in accordance with fluid mechanics standards and the flow through the orifice calculated from a knowledge of the static head and the orifice coefficient.

Several different velocity profiles are shown in Fig. 5. The slope of the velocity with distance curve, $d\bar{u}/dy$, gives a shear rate related to the classical definition of laminar shear in the system. The maximum shear rate is the maximum slope of the velocity profile across the jet width. This is given in Fig. 7. The maximum shear rate is proportional to the impeller speed for the 6-in turbine and is roughly proportional to the impeller diameter at constant speed, as shown by points for the 4-, 6- and 8-in turbines. Table 3 gives a summary of these data points.

The average shear rate is the average of the various shear rates across the jet profile and is also plotted in Fig. 7. These shear rates are also proportional to impeller speed. The 4- and 8-in impellers gave approximately the same shear rate as the 6-in at the same impeller speed. This can be understood, since the maximum centreline velocity for a turbine is proportional to the impeller diameter at constant speed, while the width of the blade is also proportional to the impeller diameter giving approximately the same average shear rate at a given speed, regardless of the impeller diameter.

In addition to these considerations of mean velocity gradients with distance across the jet, there is the whole picture of turbulent flow patterns. The electric amperage reading from the hot-wire probe was passed through a root mean square analyser which gave the root mean square of the fluctuating velocity component, u', which is related to the intensity of turbulence in the system. The root mean square values for several speeds for a 6-in impeller is plotted in Fig. 6.

A plot of the root mean square value over the average velocity, $\bar{u}$, at that point gives another indication of the actual width of the high-velocity jet stream from the turbine, as shown in Fig. 8.

Heat transfer

Heat transfer is an important part of most mixing operations. Published data on the heat transfer to helical coils deal primarily with flat-blade turbines. Experiments made in the same fashion as reported in the paper by Oldshue and Gretton[3] were carried out with marine-type three-blade square-pitch propellers (Fig. 9).

It was found that the exponents on any given quantity were the same for the propeller as for the flat-blade turbine so that the heat transfer equation is given as follows:

$$\left(\frac{h_o d}{k}\right) = 0{\cdot}091 \left(\frac{ND\rho}{\mu}\right)^{0\cdot67}\left(\frac{C_p\mu}{k}\right)^{0\cdot37}\left(\frac{D}{T}\right)^{0\cdot1}\left(\frac{d}{T}\right)^{0\cdot5} \quad \ldots\ldots (2)$$

$$h_h \text{ or } h_c = h_o(\mu_s/\mu)^{-m} \quad \ldots\ldots (3)$$

Fig. 10 shows the experimental correlations.

At a given Reynolds number, the propeller draws less horsepower than does a flat-blade turbine so that it has a lower heat transfer coefficient (Table 5). However, the heat transfer coefficient of the propeller at constant power input and impeller diameter is approximately the same as for the flat-blade turbine.

This means that the propeller having higher speed for operation would require less torque for a given installation. It was found that the impeller positioning from $C/D = 0{\cdot}33$ to $C/D = 1{\cdot}67$ for a propeller at constant power had no effect on the heat transfer coefficient. This seemed to be due to the fact that the axial flow pattern of the propeller in a baffled tank is about the same, regardless of the propeller position in the system.

In contrast to the work on the flat-blade turbines, the propellers are somewhat more sensitive to baffle position. Baffles inside the coil bank gave 15% higher coefficients than outside the coil bank, apparently because there was a greater swirling action between the coil and the tank wall which gave a higher heat transfer coefficient.

Table 5. Heat Transfer Coefficient for Propeller Compared with Turbine

$h_{o\,prop}/h_{o\,fbt}$ *at constant* N_{Re}	HP_{prop}/HP_{fbt} *at constant speed*	$h_{o\,prop}/h_{o\,fbt}$ *at same diameter and constant power*
0·54	0·06	1·0

$N_{Re} > 10{,}000$

Table 6. Properties of a Fluid Mixer on Scale-up

Property	*Pilot scale 5 gal*	*Plant scale 625 gal (geometric similarity)*			
P	1·0	125	3,125	25	0·2
$P/vol.$	1·0	**1·0**	25	0·2	0·0016
N	1·0	0·34	1·0	0·2	0·04
D	1·0	5·0	5·0	5·0	5·0
Q	1·0	42·5	125	25	5·0
$Q/vol.$	1·0	0·34	**1·0**	0·2	0·04
ND	1·0	1·7	5·0	**1·0**	0·2
$ND^2\rho/\mu$	1·0	8·5	25·0	5·0	**1·0**

Scale-up

One of the characteristics of mixer scale-up is that all of the different quantities that can be used to describe the flow pattern and mixer performance in a system scale-up numerically in different fashion from a small- to a large-size system. It is therefore important to carefully consider the effect of each of these variables on scale-up and make sure that if certain minimum and maximum values on any of the quantities are required, these adjustments be done properly.

As an illustration of how several quantities change on scale-up, Table 6 shows the translation from a 5-gal pilot-scale unit to a plant-scale unit with five times the linear dimensions of the pilot unit. Quantitities that are often considered in scale-up are:

Total power, power per unit volume of liquid, impeller speed, impeller diameter, the flow from the impeller, flow per unit volume, peripheral speed and Reynolds number ($ND^2\rho/\mu$).

As an example of what happens to these various quantities when any one is fixed as a constant between the large and small size system, there are four columns holding as constants, in turn:

(1). Power per unit volume;
(2). Flow per unit volume;
(3). Peripheral speed;
(4). Reynolds number.

Referring to Table 6, it is important to consider which of the variables to hold at some particular value, and we must then allow the other ones to vary as they will.

As a more detailed example, one could consider the case of a fairly complex system which involves:

(1). Absorption of oxygen from an air stream;
(2). Chemical reaction in the liquid phase;
(3). Suspension of a catalyst in the liquid;
(4). Heat transfer in the system.

For this example, assume that the chemical reaction is fast, so that the dissolved oxygen level in the system approaches zero and the process rate is governed by the mass transfer from gas to liquid. Assume, also, that the liquid/solid mass transfer step is not controlling as long as uniformity of solid suspension throughout the vessel is obtained.

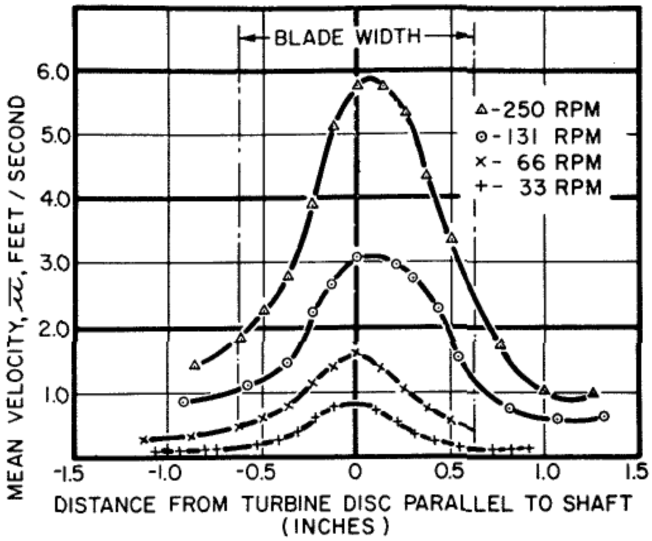

Fig. 5. Velocity profile for 6-in-d, six-flat-blade turbine, 18-in-d tank, installed as in Fig. 1. T = 18 in, D = 6 in, C = 6in, B = 1·5 in, Z = 20 in. Flow measured ¾ in away from turbine periphery

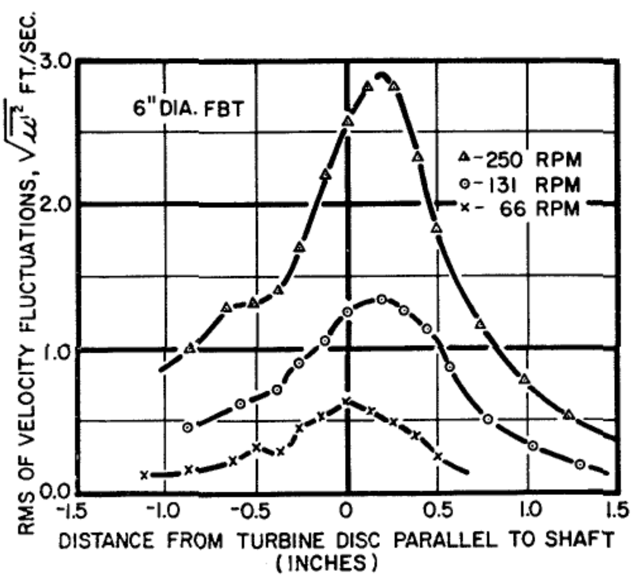

Fig. 6. Intensity of turbulence $\sqrt{\bar{u}'^2}$ from velocity meter. D = 6 in, T = 18 in, Z = 20 in, C = 6 in, B = 1·5 in, four baffles. Probe ¾ in from impeller periphery

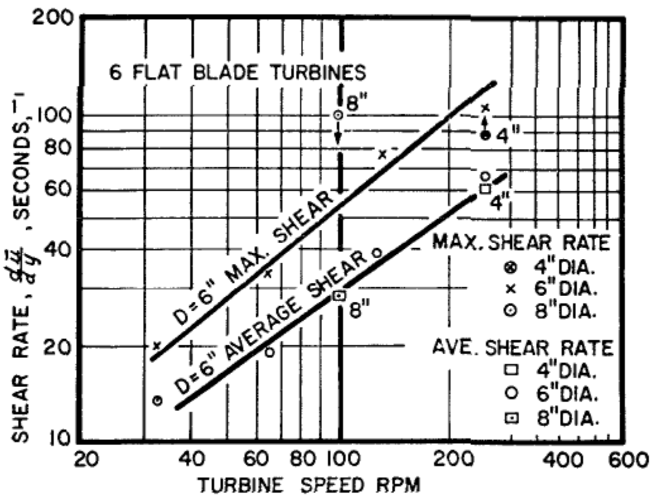

Fig. 7. Maximum and average shear rates as defined by the laminar shear definition dū/dy. T = 18 in, Z = 20 in, C = D. D = impeller d, 4, 6 and 8 in. B = 1·5 in, four in number

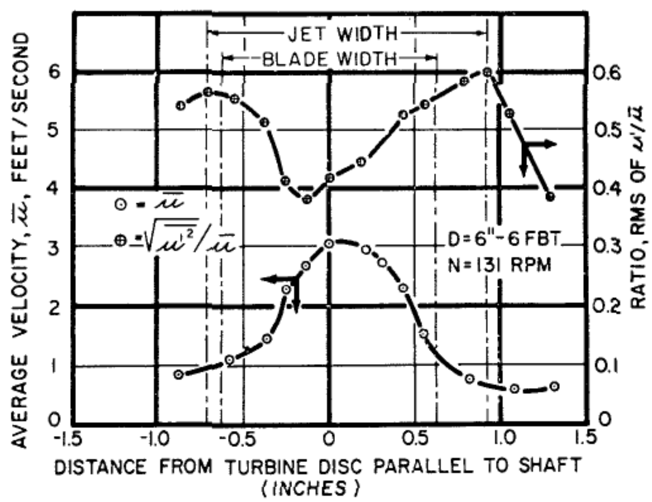

Fig. 8. Plot of ratio of rms value of velocity fluctuations to mean velocity at various points as a criterion of turbulent jet width, T = 18 in, D = 6 in, Z = 20 in, C = 6 in, B = 1·5 in, four baffles. Probe ¾ in from impeller periphery

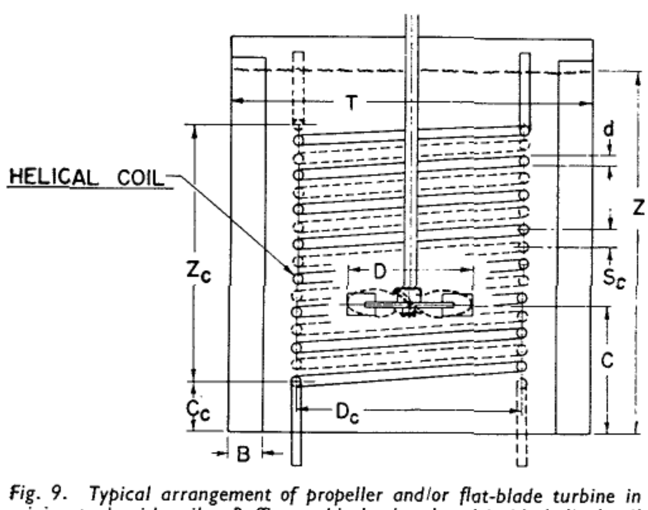

Fig. 9. Typical arrangement of propeller and/or flat-blade turbine in mixing tank with coils. Baffles could also be placed inside helical coil

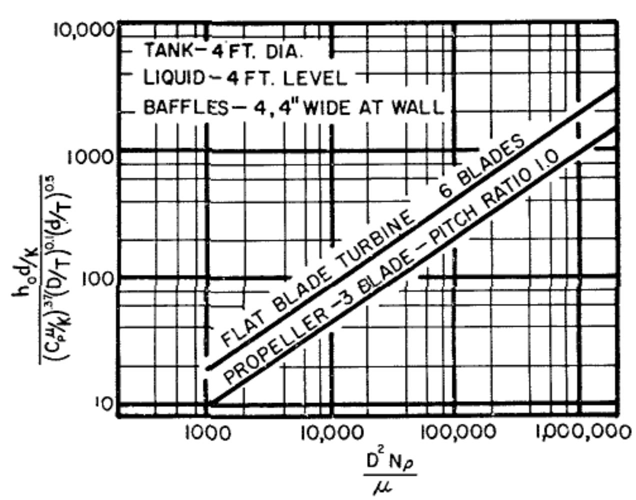

Fig. 10. Heat transfer correlation for propellers and flat-blade turbines

In the first column of Table 4 are listed the following variables in the pilot scale:

Tank diameter, impeller horsepower, impeller diameter, number of turbines, liquid depth, volume of tank (gal), air flow (ft³/min), superficial velocity, F (ft/s), based on empty tank cross-section, total heat load, temperature difference, area for heat transfer (ft²), overall coefficient, U (Btu/h ft² °F).

For purposes of calculation we may either use the absolute values of the quantities or ratios. For the gas/liquid mass transfer, the units for Δ_c are ppm of oxygen; the units of K_La are h^{-1}, and the units of the mass transfer rate are ppm/h.

To compare laminar shear rates and turbulent shear intensity, ratios are used. Arbitrarily, the maximum laminar shear in the pilot unit in the turbine jet is assigned a value of 1·0. All the other values for laminar shear rate are consistent with this value of 1·0.

Again arbitrarily, the turbulent shear intensity in the pilot unit in the turbine jet is given a value of 10·0. There is no numerical relationship between the numerical value chosen for the turbulent shear intensity and the numerical value chosen for laminar shear rate.

Other considerations shown in Table 4 are the power actually used in the tank compared to the minimum power required for complete uniformity of the solids and the flow from the turbine per unit volume of the tank.

In the second column of the table, a trial is made considering the requirements to give the proper mass transfer rate with a 75% absorption of oxygen from the air stream. Due to the increased absorption of oxygen, the partial pressure of oxygen in the exit gas is lower. However, the total pressure at the bottom of the tank is higher, yielding a log mean, Δ_c, driving force of 7·4 compared to 9·3 ppm previously. This means that a K_La value of 1·25 times higher is required. The volume of gas required in the large-size system gives a superficial velocity which is higher than it was in the pilot unit. This means that the power level in the plant unit can be lower. In this column, geometric similarity has been maintained.

Considering the other variables, the maximum laminar fluid shear has gone up, while the average laminar shear has gone down. In the large-size unit, the flow per unit volume is also less. However, the solid suspension requirements are still met adequately.

Looking back over the laboratory data, it is felt that the maximum shear rate should be lowered to about 45% above that in the pilot unit to avoid any possible breakdown of the catalyst particles. Therefore, geometric similarity is changed and an impeller-size to tank-size ratio of 0·46 is selected for use in column 3. This requires a higher horsepower, however, for the mass transfer step, since this is going away from the optimum D/T ratio. A new horsepower is shown and this change to a larger diameter lower-speed unit changes the other variables as shown in the table. This is adjudged to be satisfactory for full-scale operation and the mechanical design of the entire unit can then proceed.

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[4] J. P. Sachs and J. H. Rushton, *Ibid.*

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