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FERMENTATION MIXING SCALE-UP TECHNIQUES

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Fermentation Mixing Scale-Up Techniques

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Summary

Scale-up of a mixing process in fermentation involves breaking the process down into individual but interrelated steps. The effect of mixing on gas–liquid absorption, fluid shear rates, blending, and heat transfer allows each to be considered separately. A method of estimating the mixer size required in a full-scale system from pilot studies illustrates the application of a particular technique.

THE PROBLEM OF SCALE-UP

In scaling up a fermentation process, one is dealing with a net result from several independent, but interrelated, steps. A mixing vessel is three-dimensional, so that as the linear dimensions increase, the capacity of the system increases as the cube of the linear dimension. With this increase in scale, other variables rise on the linear scale with different exponents, which may vary from negative to zero to three and higher. There are several techniques used to scale up which vary in complexity and detail.

The problem is to break down the fermentation process, with sufficient detail that the individual steps are manageable in terms of data and theory, and can also shed light on the basic engineering parameters involved.

To illustrate the problem, first consider the case of a batch-mixing operation in a single- or multiphase fluid.

From Table I, it is seen that with a pilot unit of 20-gal. capacity, and with some of the parameters thought to be important assigned to an arbitrary value of 1.0, upon scale-up to a 2500-gal. tank (maintaining geometric similarity) it is impossible to keep all the parameters in the same ratio to one another. Even here it is necessary to pick and choose which ones will be controlled, and then examine what effect the ones that cannot be controlled have on our overall result. For

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*Manufacturer of Lightnin Mixers and Aerators example, a particular method of scale-up may be chosen only to find that the Reynolds number has increased tenfold. Most of the time, from a process standpoint, this increase is academic, but to get the correct power number for the impeller, this could be quite important.

TABLE I
Properties of a Fluid Mixer on Scale-Up

Property	Pilot scale, 20 gal.	Plant scale, 2500 gal.			
P	1.0	125	3125	25	0.2
P/V	1.0	[1.0]	25	0.2	0.0016
N	1.0	0.34	1.0	0.2	0.04
D	1.0	5.0	5.0	5.0	5.0
Q	1.0	42.5	125	25	5.0
Q/V	1.0	0.34	[1.0]	0.2	0.04
ND	1.0	1.7	5.0	[1.0]	0.2
$ND^2\rho/\mu$	1.0	8.5	25.0	5.0	[1.0]

In general, there is no basis for saying that any particular group of parameters should remain constant in scale-up, unless it can be demonstrated, theoretically or experimentally, that in this "particular" mixing application it is constant.

At this point, it would be well to digress and classify the entire field of fluid mixing as an aid to the discussion which follows.

Referring to Table II, it is seen that the field of mixing can be broken down into the area of process design, the area of power *consumption characteristics, and the area of mechanical design.*

TABLE II
Elements of Mixer Design

Process Design	Fluid mechanics of impellers Fluid regime required by process Scale-up; hydraulic similarity
Impeller power characteristics	Relate impeller HP, speed, and diameter
Mechanical design	Impellers Shafts Drive assembly

Several of these areas will be discussed. For process design, the first consideration is that the mixer must be designed to accomplish the process objective. After the geometry and the type of impeller have been chosen, then only two independent choices remain, (any two of the three variables: power, diameter, and speed). This is true because the relationship developed for power characteristics holds true for any mixing process and, incidentally, whether the process is accomplished satisfactorily or not.

A study of the fluid mechanics in a mixing vessel is often done without regard to any particular mixing requirement. This is quite desirable, since the decision whether a mixing flow pattern is good or bad depends upon the process requirement; because these requirements are almost limitless, there is great need for a basic understanding of various types of mixing flow patterns, so that the proper pattern may be chosen on the proper occasion.

To treat the fluid regime requirements of mixing processes, the field may be broken up into at least five categories of the types of liquids and solids present (Table III). There are two basic divisions, depending upon whether the mixing criterion is a quantitative chemical reaction or mass transfer rate, or whether the criterion involves uniformity of concentrations of liquids, gases, or solids.

Within each of these ten areas, there is actually a separate mixing technique, with dependence on the basic mixing principles underlying all processes.

TABLE III
Mixing Processes

Physical criteria	Application classes	Mass transfer criteria
Suspension	Liquid–solid	Dissolving
Dispersion	Liquid–gas	Absorption
Emulsification	Immiscible liquids	Extraction
Blending	Miscible liquids	Reaction
Pumping	Fluid motion	Heat transfer

Gas–Liquid Scale-Up

The particular problem in scaling up a gas–liquid operation typical of a fermentor must now be examined. Figure 4 indicates some of the parameters involved. In this case, the linear gas velocity,

TABLE IV
Gas–Liquid Scale-Up at Constant Volumetric Absorption Rate

Property	Pilot scale, 20 gal.	Plant Scale, 2500 gal.			
T	1.0	5.0	5.0	5.0	3.5
Z/T	1.0	1.0	1.0	1.0	2.8
V_g/V/min.	1.0	1.0	0.4	0.2	0.1
F, ft./sec.	0.1	0.5	0.2	0.1	0.1
Gas absorption efficiency, %	x	x	$2.5x$	$5x$	$10x$
K_La required	1.0	<1.0	>1.0	≫1.0	>>>1.0

usually expressed as a superficial velocity based on the empty tank cross section, is an important factor in the design of the reactor, in terms of the energy the gas stream adds, the mixing energy it requires to overcome and disperse this gas stream, and the physical appearance of the tank (in terms of surface turbulence and foaming).

Referring to Table IV, the major problem can now be seen. If it is necessary or desirable to maintain equal volumes of gas per volume of liquid per minute then the linear gas velocity through the vessel increases almost directly with scale ratio. However, if the percentage of active gas absorbed on the smaller scale is relatively small, it is possible to consider reducing the volume of gas per volume of liquid per minute, and to let gas absorption efficiency increase. Four cases are shown in Table IV. The last case shows the effect of a geometry change.

The pumping capacity of the impeller and the circulation through the tank is fairly easy to visualize, and there has been considerable study of these circulation rates. However, the other component involved in a mixing system, fluid shear, is somewhat more elusive. In fact, it is necessary to distinguish very carefully between fluid shear rate and fluid shear stress.

The scale and frequency of the fluid shear rates must be taken into account. The general principle is that the only fluid shear rates that need be considered, in any particular aspect of the mixing process, are those which have a scale and frequency such that they are effective with the particle or the bubbles under consideration. One does not use an ultrasonic drill to drive a rivet, nor a riveting hammer to drill a tooth cavity.

Maintaining the objective of breaking down the process mechanism only so far that it is manageable, some of the individual components of the fermentation process will be examined to see what can be learned, in quantitative and qualitative ways, that will help solve an actual design requirement.

Before going any further, it might be well to reassure those who feel that the problem is insolvable, and that it is a wonder that anything was ever scaled up. Many, many processes are quite insensitive to the detailed nature of the fluid flow pattern of the system, and the problems of concern here are of little consequence. Experience and study have yielded useful parameters, and, from a practical standpoint, no further breakdown is needed. Many operations may be studied by examining the overall result on two different, but relatively small, scales, and excellent scale-up relationships may then be derived. However, the particular case of fermentation, in which living bacterial solids are affected by fluid shear, and in which high mass-transfer rates require large expenditures on equipment and power, has all the ingredients necessary to make a thorough study of the mixing mechanisms important and rewarding.

For the purpose of this discussion, the fermentation process may be broken down into the following steps:

1. Gas–liquid mass transfer.
2. Fluid shear, related to the biological solids.
3. Blending.
4. Solid suspension.

MIXING PARAMETERS OF THE INDIVIDUAL STEPS

Fluid Shear Rates

When the question of fluid shear rates comes up, a quick way to decide whether the discussion is worthwhile or not is to raise the following questions:

1. What is the scale and frequency of the shear stresses which would be involved?

2. What is the size and density of particles or bubbles involved?

3. Are there any data to help evaluate the effect after the objectives have been defined?

Some data may now be examined which will give a qualitative view of the flow from a turbine impeller. If the objective were to find the mean velocity at a given point, then some modification of a pitot tube would be the most practical method. However, it was desired to determine the rapid fluctuations in a stream, and so the hot-wire velocity meter was used, as shown in Figure 1. The primary purpose here is to give the magnitude and ratios of some of the effects, and to avoid becoming involved in complicated mathematical discussions of flow and turbulence. For simplicity, only the measurements made when the probe was positioned as shown in Figure 1 will be considered here.

The pumping capacity of flat-blade turbines has been measured by various investigators. The flat-blade turbine referred to here is of the type shown in Figure 2. It has six blades, and the dimension ratios given on the figure.

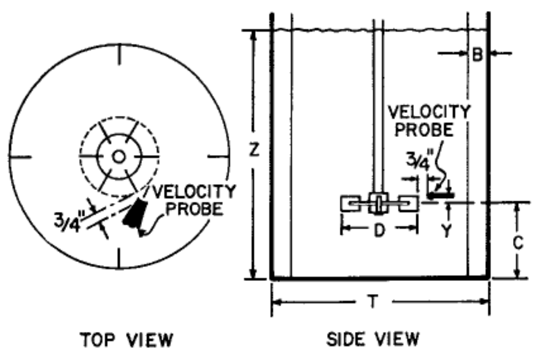
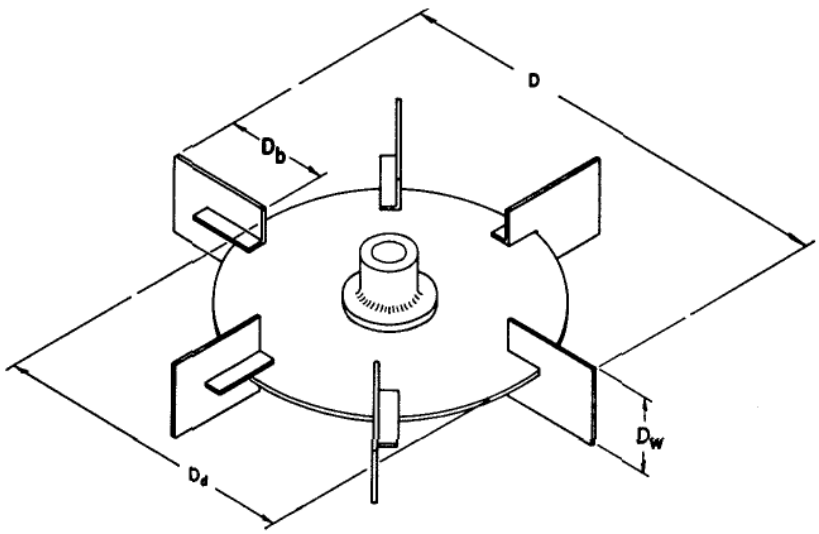


Fig. 1. Typical turbine installation in baffled tank showing location of hot-wire velocity probe.

One of the most complete works on pumping capacity is that of Sachs,[1] who used the photographic technique of analysis for velocity patterns. To illustrate the type of flow pattern, Figures 3 and 4 are included from his work. They show the overall type of flow developed by a flat-blade turbine.

To obtain more information on the nature of flow from a turbine impeller, a hot-wire velocity meter was developed for use in mixing systems. The measuring element consists of a small wire suspended between two electrodes. The wire was 0.100 in. long and 0.0007 in. in diameter. The wire was heated electrically to about 20°F. above the tank temperature. As the water flows by the probe, it tends to



$D : D_b : D_w : D_d = 1 : \frac{1}{4} : \frac{1}{5} : \frac{2}{3}$

Fig. 2. Typical dimensions of flat-blade turbine impeller.

cool it, and thus the amount of current required to maintain constant temperature is related to the velocity of the fluid.

The electronic arrangement of the equipment is as described by Hubbard.[2] A hot-wire probe gives the maximum flow rates when the wire is at right angles to the flow stream. In this study, the wire was always positioned in a horizontal plane and was rotated at various angles to a radius from the centerline of the tank until maximum velocity readings were obtained. From this study, it appeared that the angle for maximum velocity corresponded to a tangential velocity component at the impeller periphery.

In the data reported here, the probe was positioned $^3/_4$ in. away from the turbine periphery, directed along a tangent to the impeller periphery, and was positioned at various distances above and below the turbine centerline.

The tank diameter, T, used in this work was 18 in., and the liquid level, Z, was 20 in. Impeller diameters, D, used were 4, 6, and 8 in. Four baffles were used in the tank, each 1.5 in. wide.

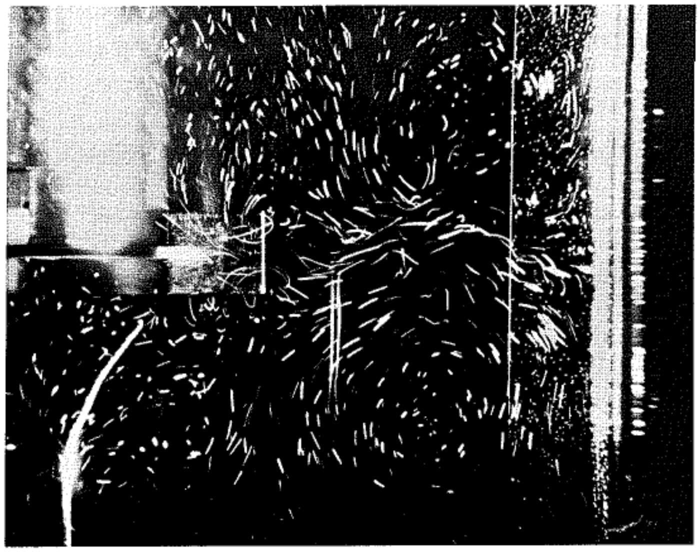


Fig. 3. Flow pattern, side view, 4-in diameter flat-blade turbine, 12-in. diameter tank, water. Streak photography showing particle velocities in a $^1/_4$-in. wide vertical plane.

The hot-wire velocity meter has a very fast response, so that the velocity, u, at a point at any time can be expressed as a mean velocity over a time interval, $\bar{u}$, plus a fluctuating velocity component, u', so that

$$u = \bar{u} + u' \tag{1}$$

The probe was calibrated by placing it into the flow from an orifice in a tank with a constant head for the period of the calibration. The orifice was constructed very carefully in accordance with fluid mechanics standards, and the flow through the orifice calculated from a knowledge of the static head and the orifice coefficient.

A typical trace from a hot-wire probe is shown in Figure 5. This is *not* an actual recording from this experiment.

Figure 6 shows the results at three different speeds as the probe was moved up and down from the horizontal plane through the impeller center.

Table V lists flow characteristics determined with a hot-wire velocity meter.

Fig. 4. Typical flow pattern from flat-blade turbine in baffled tank, bottom view, 4-in. diameter turbine, 12-in. diameter tank, water; $^1/_4$-in. wide horizontal plane in impeller centerline.

TABLE V
Flow Characteristics as Measured with a Hot-Wire Velocity Meter

D, in.	Impeller speed, N, rpm	Maximum shear rate, $d\bar{u}/dy$, sec.$^{-1}$	Average shear rate, $d\bar{u}/dy$, sec.$^{-1}$	Mean rms value, $(\bar{u}'^2)^{1/2}$ ft./sec.
4	257	88.5	62.4	1.02
6	66	32.3	10.4	0.42
6	131	78.7	39.3	0.97
6	250	106.2	65.0	1.86
8	100	100.0	29.3	1.01

Rate and Stress

The shear rate in a system is the velocity gradient between adjacent layers. The product of viscosity and shear rate gives shear stress, as shown by the following equations:

$$\text{Shear rate} = \Delta u/\Delta y = du/dy \quad (2)$$

$$\text{Shear stress} = \mu(\text{Shear rate}) \quad (3)$$

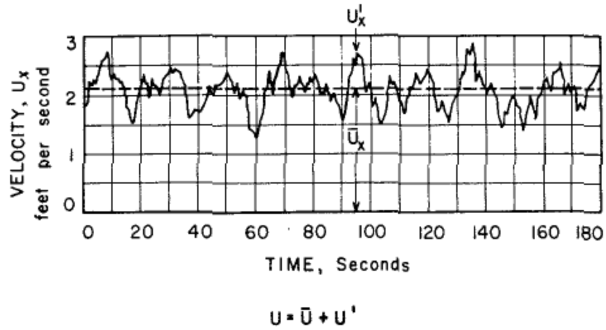


Fig. 5. Typical velocity from hot-wire velocity probe. This is not a chart from this experimental program.

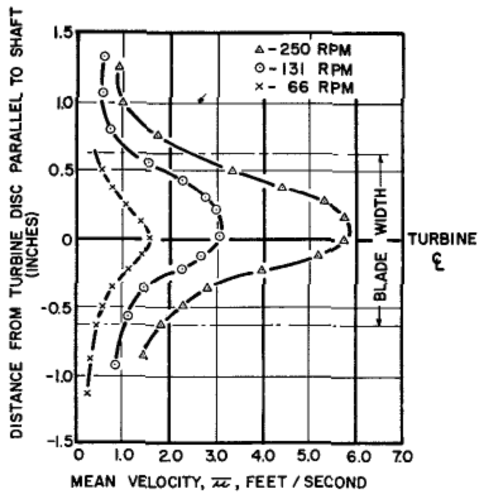


Fig. 6. Mean velocity, μ, at various positions above and below a 6-in. diameter turbine, 6 flat blades.

The average value of the slope of the mean velocity, $\bar{u}$, vs. distance line is plotted in Figure 7. The maximum slope measured is also plotted in Figure 7. With the 6-in. flat-blade turbine, the maximum shear rate was approximately twice the average shear rate. However, as the impeller size was decreased, or increased, the average shear rate remained about the same at the same impeller speed

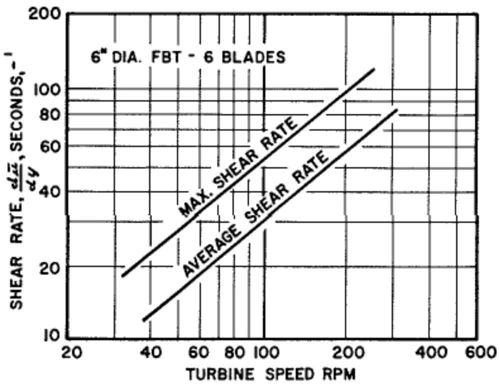


Fig. 7. Maximum and average shear rates obtained from Fig. 6.

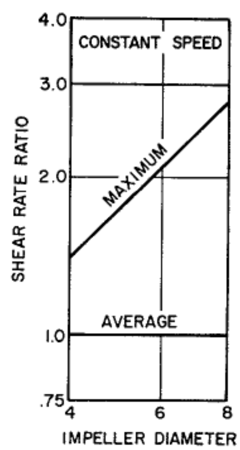


Fig. 8. Ratio of maximum and average shear rates for 4, 6, and 8-in. flat-blade turbine impellers, compared to average shear rate for 6-in. diameter impeller.

(Fig. 8). The maximum shear rate increased with impeller diameter at the same impeller speed. This indicates that, as scale-up progresses, lower average shear rates will be encountered in the stream from the impeller due to lower impeller speeds, but higher maximum shear rates, due to the higher peripheral velocity of the impeller.

The next question, then, is: What kinds of velocity fluctuations are found in this stream?

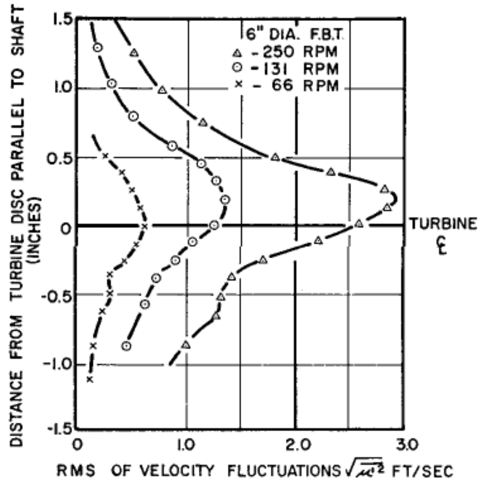


Fig. 9. Rms of velocity fluctuations for 6-in. diameter flat-blade turbine as shown in Fig. 6.

A little verse which serves to remind us of what happens in a turbulent system is:

Big size whirls have little whirls
That feed on their velocity;
And little whirls have lesser whirls
And so on to viscosity.

By taking the root mean square of the velocity fluctuations, one obtains a measure of the intensity of the turbulent fluctuations. This particular probe (and electronic equipment) considers all fluctuations having a frequency of 100 cps or higher.

To obtain a complete picture of the turbulent fluctuations and energy decay and dissipation, it would be necessary to actually analyze the complete spectra of frequency and size of the turbulent currents.

All the horsepower applied to the tank appears as heat, and since this is generated by viscous shear, when the eddy becomes small enough, its energy is dissipated by that mechanism. Therefore, processes that occur at the molecular level may well be a function only of energy input per unit volume.

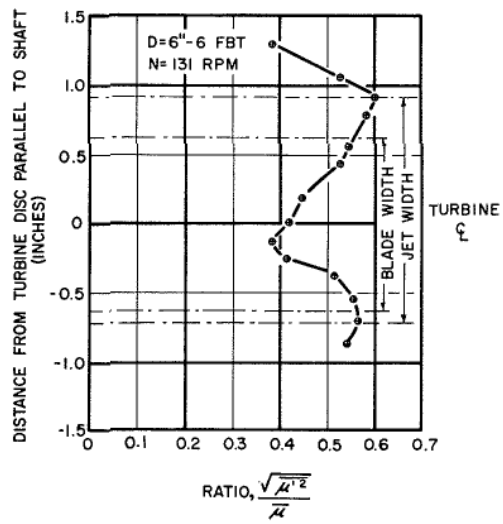


Fig. 10. Ratio of rms velocity fluctuations to mean velocity at corresponding points.

As can be seen from Figure 9, the intensity of turbulence follows a pattern similar to the mean velocity. The ratio of the root mean square of the fluctuations to the mean velocity is between 0.4 and 0.6 within the jet from the impeller (Fig. 10).

If it were known what each step of the fermentation process required in the way of shearing action, then it would be possible to carry out a relatively rigorous calculation throughout the system. An example of the length to which this can be carried will be given in the concluding section.

Viscosity

Viscosity affects the terminal velocity of rise for air bubbles. Table VI shows the various bubble sizes required to have various terminal rise values for materials with a specific gravity of 1.0 and varying viscosities. If air bubbles are to rise from viscous materials with any appreciable velocity, then they must be of fairly good size. In material of 1000 cp. viscosity, bubbles that are less than $^1/_2$ in. in diameter will remain in the system, and only those that are $^1/_7$ in. or larger will enter and pass out with any appreciable velocity.

TABLE VI
Effect of Viscosity on Required Bubble Diameter for Various Terminal Rise Velocities

Viscosity cp.	Bubble diameter, in. Terminal rise velocity, ft./min. 60 ft./min.	10 ft./min.	1.0 ft./min.	0.1 ft./min.
1.0	0.04	0.02	0.005	0.002
10	0.09	0.04	0.01	0.004
100	0.2	0.1	0.03	0.01
1,000	0.6	0.3	0.08	0.03
10,000	2.0	0.6	0.2	0.06
100,000	4.0	2.0	0.5	0.2

This, of course, causes much more difficulty on a small-scale pilot-plant experiment than it would on a large-scale fermentor.

Effect of Impeller Diameter/Tank Diameter Ratio on Gas–Liquid Mass Transfer

If the above variables were not sufficient, it was found that the optimum impeller-to-tank size ratio for gas–liquid mass transfer depended on the input of mixing power relative to the gas velocity to the system. At relatively low levels of mixer horsepower (Fig. 11),

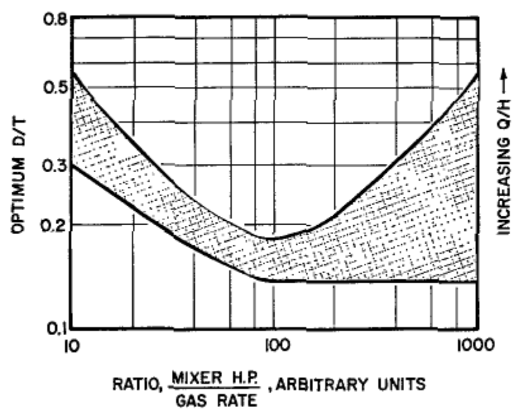


Fig. 11. The shaded area in the center shows the optimum ratio of impeller size to tank size for various ratios of mixer horsepower to gas flow. This latter ratio is in completely arbitrary units.

it is seen that relatively large D/T ratios are needed in order to have sufficient impeller pumping capacity to handle the volume of gas passing by the impeller. As the amount of power to the mixer is increased, the optimum D/T is reduced to a relatively small value, in the vicinity of 0.15 to 0.2. As the mixer power is increased far beyond that needed to disperse the gas, D/T becomes unimportant because it makes very little difference how the power is distributed.

Geometry

The geometry of the impeller and tank has a large effect on the gas–liquid mass transfer coefficient, but the coefficient also depends upon what ratio of mixer horsepower to gas rate we have in the system. Five configurations (Fig. 12) were used at $D/T = 1/3$ and at a range of horsepower levels typical of industrial practice for a superficial gas velocity of 0.1 ft./sec. These data (Fig. 13, Table VII) indicate that, within fairly large ranges of geometry variables, a similar mass transfer coefficient is obtained if the power per unit volume is maintained at similar values. This indicates that, under these conditions, there is an averaging process taking place, so that a lesser dispersion from three turbines is quite similar (in total) to a more intense dispersion from two impellers. All of these im-

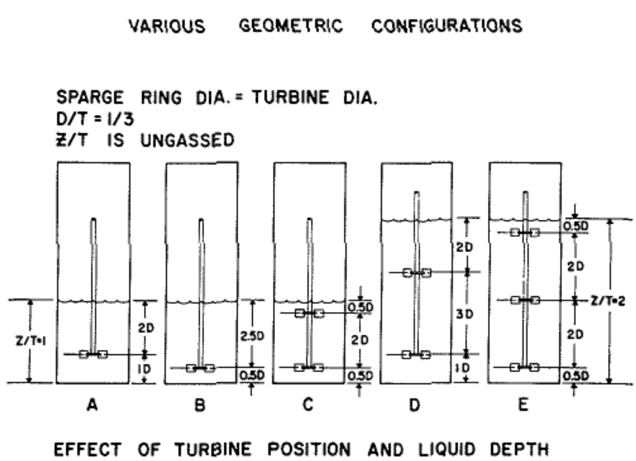


Fig. 12. Geometry used for the data recorded in Table VII and Fig. 13.

TABLE VII

Effect of Impeller Position and Liquid Depth[a]

No. of turbines	Liquid depth, in.	Run No.	Impeller position: Lower off bottom	Impeller position: *Middle* spacing	Impeller position: *Upper* spacing	Impeller position: Cover, in.	rpm	*HP* per 1000 gal.	K_Ga
1	12	139	4			8	568	2.62	0.044
		140					835	8.32	0.99
		141					1050	16.25	0.134
		142					1400	29.50	0.188
1	12	134	2			10	575	2.08	0.041
		135					793	5.81	0.085
		136					1028	10.90	0.125
		137					1218	16.32	0.171
2	12	130	2		8	2	429	2.88	0.044
		131					525	4.61	0.061
		132					650	7.59	0.084
2	24	107	4		12	8	615	3.79	0.056
		108					820	8.44	0.103
		109					1088	18.93	0.159
3	24	116	2	10	10	2	459	2.75	0.037
		117					648	6.35	0.073
		118					788	10.78	0.119

[a] Tank: 12 in. diam., dish bottom, four 1-in. baffles; turbine: 4 in. diam., 6 flat blades; air: 0.1 ft./sec.

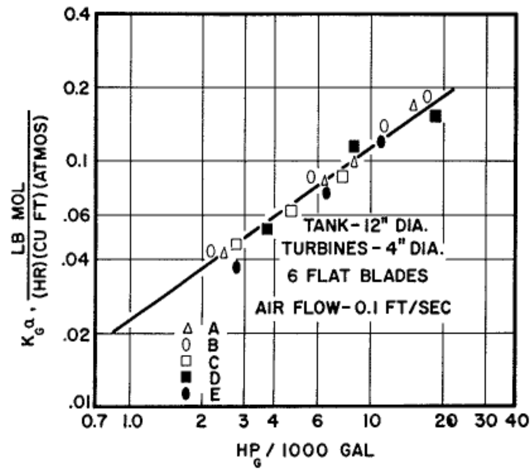


Fig. 13. Effect of impeller and tank geometry. Symbols refer to Fig. 12.

peller positions are well within the range considered acceptable from an overall fluid motion and blending standpoint. In most cases, the range of good industrial practice would allow some latitude in the spacing and placing of impellers, and this can be done to accommodate other variables in the system.

Gas Velocity

The gas passing through the mixing system changes the power drawn by the impeller at a given speed. If the absorption rate is measured for various gas rates, it can be correlated at constant speed or at constant horsepower. These two correlations could typically differ by as much as is shown on Figure 14. These different methods of correlation account for some of the variations in the effect of gas rate found in the literature.

Blending and Solid Suspension

Blending and solid suspension are usually not problems in fermentation because of the normally high power levels required for gas–liquid mass transfer. Blending is not necessarily related to flow per unit volume in the tank upon scale-up, because it also depends upon the shear rate and turbulence levels in these streams.

In fact, trying to maintain constant flow per unit volume on scale-up is almost an impracticality.

Solid suspension, *per se*, is usually not a problem in fermentation, since the settling velocity is so low. Fermentation solids contribute, mainly, a pseudoplastic nature to the fluid. However, if there were particles that had settling velocities above 1 ft./min., solid suspension

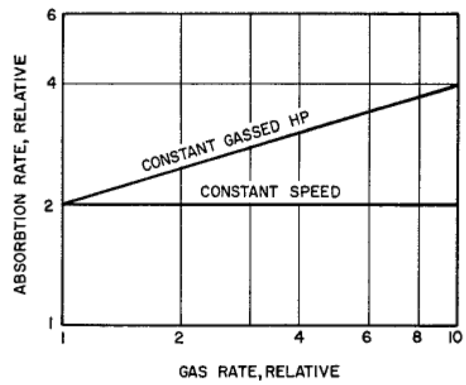


Fig. 14. The effect of gas rate is different depending upon whether correlating at constant impeller speed or constant gassed horsepower.

characteristics could be considered in detail. There are many industrial processes not involving fermentation where solid suspension requirements are equally as important as the mass transfer requirements.

AN EXAMPLE OF FERMENTATION SCALE-UP

A fermentation has been conducted satisfactorily in the pilot plant in a tank 18 in. in diameter, with an 18-in. ungassed liquid level. A single flat-blade turbine impeller, 6 in. in diameter and 6 in. from the bottom of the tank, was used. Of all the conditions tested, it was decided that an airflow of 2 ft.^{3}/min. and an impeller horsepower of 0.3 were to be used for scale-up. Under these conditions, 4500 Btu/hr. of heat were evolved.

It was decided to explore a full-scale fermentation unit in a 12-ft. diameter tank containing 25,000 gal. of ungassed fermentation broth at the initial stage (Fig. 15, Table VIII).

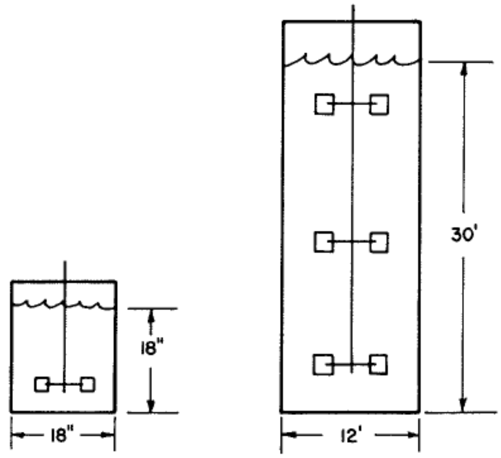


Fig. 15. Schematic illustration of scale-up example given in Table VIII.

The first consideration is to make the gas–liquid mass transfer rate equal to what it was in the pilot plant. It was estimated that the dissolved oxygen level at the most extreme condition for mass transfer was 6.0 ppm in the pilot plant, and should be 8.0 ppm in the full-scale plant. The average saturation dissolved-oxygen value at the temperature of the broth was 7.0 ppm at 1 atm. The mass-transfer rate was estimated to be 2400 ppm/hr. This is a relative term, and does not need to correspond numerically to the actual uptake value of the system.

In the larger system, the driving force is 4.2 compared to 3.7, because of the increased pressure due to the liquid level, so that the mass transfer coefficient, K_La, can be lower. Based on this mass transfer coefficient, the horsepower for the large installation was estimated at 300.

At this point, the ratio for laminar shear rates (maximum and average) can be calculated, as shown in Table VIII. The ratios of turbulent shear intensity and the total energy dissipation are also shown.

It was believed that the only shear rate of concern to the organism is the maximum shear rate in the turbulent jet. In the second column, it is seen that this has risen to 1.8 times the value of the pilot plant. In order to reduce this value, a nongeometric scale-up is considered.

TABLE VIII
Scale-Up of a Mixing Process

Properties		Plant scale: Design to satisfy mass transfer	Plant scale: Design to reduce fluid shear rate
Tank			
T, tank diam.	18 in.	12 ft.	12 ft.
P, impeller hp	0.3	300	350
D, impeller diam.	6 in.	48 in.	60 in.
No. of turbines	1	3	3
Z, (liq. level, ungassed)	18 in.	30 ft.	30 ft.
V, gal.	20	25,000	25,000
Air flow, Cfm	2.0	1400	1400
F, ft./sec.	0.02	0.15	0.15
Total heat, Btu/hr.	4500	5,650,000	5,650,000
Δt, °F.	40	80	80
Area for heat transfer, ft.^{2}	0.75	565	565
U, Btu/hr.$^{-1}$/ft.^{2} °F.	150	125	125
Process			
Mass transfer rate ($K_La \times \Delta c$), ppm/hr.	2400	2400	2400
K_La, hr.$^{-1}$	650	570	570
c^*, ave.	9.7	12.0	12.0
Δc	3.7	4.2	4.2
% Active gas absorption	2	3.5	3.5
Laminar shear rate[a]			
In turbine jet, max.	1.0	1.8	1.4
In turbine jet, ave.	0.5	0.22	0.13
Turbulent shear intensity[a]			
In turbine jet, ave.	10	18	14
Energy dissipation	1.0	0.75	0.9
Flow from impeller per tank volume	19	4	7
HP above minimum for complete suspension	10	2.0	2.5

[a] These are ratios only. There is no numerical correspondence between laminar shear rates and turbulent shear intensity.

From the results of pilot-plant work, it was felt that a maximum shear rate 40% higher than that existing in the pilot plant would still be acceptable. A 60-in. diameter turbine was next chosen. Because this particular D/T was not as effective for gas–liquid mass transfer at this particular combination of horsepower and gas flow, the horsepower was increased to 350. This allowed the shear rates to drop to the value shown in the third column, and was further characterized by a decrease of the blend time in the system, and also by an increase of the horsepower above the minimum for complete suspension.

There are many other variables that could be calculated, and many other assumptions that could be made for some of the variables that were set. If pilot-plant work were being done with this type of scale-up in mind, then there are many experiments in the pilot plant which could indicate quite precisely the data needed for this type of scale-up.

Nomenclature

- B Baffle width
- Btu British thermal units
- C Impeller distance off tank bottom
- C/D Off-bottom-distance to impeller diameter ratio
- Cfm Cubic feet per minute of air flow
- c_s Saturation concentration of dissolved oxygen at a given temperature with air
- c^* Saturation dissolved oxygen concentration corresponding to average oxygen partial pressure in gas phase
- D Impeller diameter
- D_b Turbine blade length
- D_d Disk diameter
- D.O. Dissolved oxygen
- D_w Blade width
- D/T Impeller diameter to tank diameter ratio
- F Ft./sec.
- HP_G Horsepower, gassed
- K_Ga Mass transfer coefficient, lb.-moles/hr./ft.^{3}/atm.
- K_La Mass-transfer coefficient, hr.$^{-1}$
- N Impeller rotational speed, rpm
- $ND\pi$ Peripheral speed
- N_{Re} Reynolds number, ratio of inertia force to viscosity force, $ND^2\rho/\mu$
- P Total power
- P/V Power per unit volume of liquid
- ppm Parts per million
- Q Flow from the impeller per unit time
- Q/V Flow per unit time per unit volume of tank
- rms Root mean square
- rpm Revolutions per minute
- T Tank diameter

U	Overall heat transfer coefficient
u	Velocity
$\bar{u}$	Mean velocity over a time interval
u'	Fluctuating velocity component
V	Volume
V_g	Volume of gas
y	Height above impeller centerline horizontal plane
Z	Liquid depth
Z/T	Liquid depth to tank diameter ratio
μ	Viscosity
Δc	Average mean concentration, ppm

References

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