National
Qualifications
2026

2026 Mathematics

Higher – Paper 1 (Non-calculator)

Question Paper Finalised Marking Instructions

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General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ $\quad$ $\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$ $\quad$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\quad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\quad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)(2x+1)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	**Method 1** •1 identify common factor •2 complete the square •3 process for r and write in required form **Method 2** •1 expand completed square form •2 equate coefficients •3 process for q and r and write in required form	**Method 1** •1 $2\left(x^2+10x\ldots\right.$ stated or implied by •2 •2 $2\left(x+5\right)^2\ldots$ •3 $2\left(x+5\right)^2-47$ **Method 2** •1 $px^2+2pqx+pq^2+r$ stated or implied by •2 •2 $p=2,\ 2pq=20,\ pq^2+r=3$ •3 $2\left(x+5\right)^2-47$	3

Notes:

1. $2\left(x+5\right)^2-47$ with no working gains •1 and •2 only. However, see Candidate E.
2. In Method 2, do not penalise candidates who do not work with p, q and r.

Commonly Observed Responses:

Candidate A

$2\left(x^2+10\right)+3$

$2\left(\left(x+5\right)^2-25\right)+3$ •1 ✓ •2 ✓

$2\left(x+5\right)^2-47$ •3 ✓

See exception to general marking principle (h)

Candidate B - not using required form

$px^2+2pqx+pq^2+r$ •1 ✓

$p=2,\ 2pq=20,\ pq^2+r=3$ •2 ✓

$q=5,\quad r=-47$ •3 ^

•3 is lost as answer is not in completed square form

Candidate C

$2\left(x^2+20x\right)+3$ •1 ✗

$2\left(\left(x+10\right)^2-100\right)+3$ •2 ✓1

$2\left(x+10\right)^2-197$ •3 ✓1

Candidate D

$2\left(\left(x+10\right)^2-100\right)+3$ •1 ✗ •2 ✗

$2\left(x+10\right)^2-197$ •3 ✓1

Candidate E

$2\left(x+5\right)^2-47$ •1 ✓ •2 ✓

Check: $=2\left(x^2+10x+25\right)-47$

$=2x^2+20x+50-47$

$=2x^2+20x+3$ •3 ✓

Candidate F

$2\left(x+5\right)^2=2x^2+20x+50$ •1 ✓ •2 ✓

$\therefore 2\left(x+5\right)^2-47=2x^2+20x+3$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ express second term in integrable form $\bullet^2$ integrate first term $\bullet^3$ integrate second term $\bullet^4$ complete integration	$\bullet^1$ $\ldots + 7x^{-2}$ $\bullet^2$ $\dfrac{15x^{\frac{5}{3}}}{\frac{5}{3}}$ $\bullet^3$ $+\dfrac{7x^{-1}}{-1}$ $\bullet^4$ $9x^{\frac{5}{3}} - 7x^{-1} + c$	**4**

Notes:

1. $\bullet^1$ may be implied by the appearance of $-\dfrac{7x^{-1}}{1}$ or $-\dfrac{7}{x}$.
2. Do not penalise the appearance of '$+c$', missing integral signs or missing 'dx' at $\bullet^1$.
3. $\bullet^3$ is only available for integrating a term with a negative power.
4. Do not penalise the appearance of an integral sign and/or dx throughout.
5. Do not penalise the omission of '$+c$' at $\bullet^2$ or $\bullet^3$.
6. All coefficients must be simplified at $\bullet^4$ stage for $\bullet^4$ to be awarded.
7. $\bullet^4$ is only available for attempting to integrate both terms.
8. Accept $9x^{\frac{5}{3}} + \dfrac{7}{-x} + c$ for $\bullet^4$. However, do not accept $9x^{\frac{5}{3}} + \dfrac{7}{-1x} + c$ or $9x^{\frac{5}{3}} + -\dfrac{7}{x} + c$ for $\bullet^4$.

Question	Generic scheme	Illustrative scheme	Max mark
2.	(continued)		

Commonly Observed Responses:

Candidate A - Integrating over two lines

$\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+7x^{-2}$ $\bullet^1$ ✓

$\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}$ $\bullet^2$ ✗ $\bullet^3$ ✓

$9x^{\frac{5}{3}}-7x^{-1}+c$ $\bullet^4$ ✓$_1$

Candidate B - error in integration

$\int\left(15x^{\frac{2}{3}}+7x^{-2}\right)dx$ $\bullet^1$ ✓

$\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-3}}{-3}$ $\bullet^2$ ✓ $\bullet^3$ ✗

$9x^{\frac{5}{3}}-\frac{7}{3}x^{-3}+c$ $\bullet^4$ ✓$_1$

Candidate C - +c appears on a fully simplified line of working

$9x^{\frac{5}{3}}-7x^{-1}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$9\sqrt[3]{x^5}-\frac{7}{x}+c$ $\bullet^4$ ✓

Candidate D - simplification never fully correct

$\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$9x^{\frac{5}{3}}+\frac{7x^{-1}}{-1}+c$

$9\sqrt[5]{x^3}-\frac{7}{x}+c$ $\bullet^4$ ✗

Candidate E - +c not present on a fully simplified line of working

$\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}+c$ $\bullet^2$ ✓ $\bullet^3$ ✓

$9x^{\frac{5}{3}}-7x^{-1}$ $\bullet^4$ ^

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ state coordinates of centre	$\bullet^1$ $(-1,2)$	**4**
	$\bullet^2$ find gradient of radius	$\bullet^2$ $\frac{4}{3}$ **or** $\frac{-4}{-3}$	
	$\bullet^3$ state perpendicular gradient	$\bullet^3$ $-\frac{3}{4}$	
	$\bullet^4$ determine equation of tangent	$\bullet^4$ $4y=-3x+30$	

Notes:

1. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient.
2. At $\bullet^4$, accept $3x+4y=30$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Candidate A - unsimplified constant terms

$(-1,2)$ $\bullet^1$ ✓

$m=\frac{-4}{-3}$ $\bullet^2$ ✓

$m_{\perp}=-\frac{3}{4}$ $\bullet^3$ ✓

$y=-\frac{3}{4}x+\frac{30}{4}$ $\bullet^4$ ^

However...

$4y=-3x+30$ $\bullet^4$ ✓

$y=-\frac{3}{4}x+\frac{30}{4}$

Candidate B - $m=m_{\perp}$

$(-1,2)$ $\bullet^1$ ✓

$m=\frac{4}{3}=-\frac{3}{4}$ $\bullet^2$ ✓ $\bullet^3$ ✗

$4y=-3x+30$ $\bullet^4$ ✓$_1$ - BOD

However...

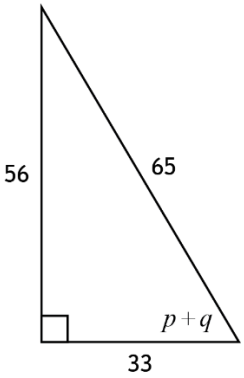
$m=\frac{4}{3}$ $\bullet^2$ ✓

$=-\frac{3}{4}$ $\bullet^3$ ✓ - BOD

$4y=-3x+30$ $\bullet^4$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] find $\sin q$	•[1] $\frac{12}{13}$	**1**
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•[2] find two remaining trig ratios	•[2] two from $\sin p = \frac{4}{5}$, $\cos q = \frac{5}{13}$, and $\cos p = \frac{3}{5}$ stated or implied by •[3] and •[4]	**4**
			•[3] select appropriate formula and express in terms of p and q	•[3] $\sin p \cos q + \cos p \sin q$	
			•[4] substitute into addition formula	•[4] $\frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$	
			•[5] find $\sin(p+q)$	•[5] $\frac{56}{65}$	
		(ii)	•[6] substitute into appropriate formula	•[6] $\frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$	**2**
			•[7] find $\cos(p+q)$	•[7] $-\frac{33}{65}$	
Notes:					

1. Accept two from $p = \cos^{-1}\left(\frac{3}{5}\right)$, $p = \sin^{-1}\left(\frac{4}{5}\right)$ or $q = \cos^{-1}\left(\frac{5}{13}\right)$ for •[2].
2. Where candidates substitute an incorrect value for $\sin p$, $\cos p$ or $\cos q$, •[4] and •[6] may be awarded if this value has previously been stated or it can be implied by a diagram or Pythagoras calculation. See Candidate A.
3. Where candidates explicitly state a value for $\sin p$, $\cos p$ or $\cos q$, subsequent working must follow from that value for •[4] to be awarded.
4. Award •[2] and •[3] for candidates who write $\sin\left(\frac{4}{5}\right) \times \cos\left(\frac{5}{13}\right) + \cos\left(\frac{3}{5}\right) \times \sin\left(\frac{12}{13}\right)$, •[4] and •[5] are unavailable.
5. For any attempt to use $\sin(p+q) = \sin p \pm \sin q$, •[4] and •[5] are unavailable.
6. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.
7. •[5] is only available for an answer expressed as a single fraction.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)	(ii)	**(continued)**		
Commonly Observed Responses:					
Candidate A - incorrect use of Pythagoras Missing sides of $\sqrt{7}$ and $\sqrt{162}$. $\bullet^1$ is unavailable. Follow through marks are available leading to: $\sin(p+q)=\frac{4\sqrt{7}+3\sqrt{162}}{13\sqrt{7}}$, $\cos(p+q)=\frac{3\sqrt{7}-4\sqrt{162}}{13\sqrt{7}}$, and $\tan(p+q)=\frac{4\sqrt{7}+3\sqrt{162}}{3\sqrt{7}-4\sqrt{162}}$				**Candidate B - incorrect use of Pythagoras** Missing sides of 5 and $\sqrt{194}$. $\bullet^1$ is unavailable. Follow through marks are available leading to: $\sin(p+q)=\frac{20+3\sqrt{194}}{65}$, $\cos(p+q)=\frac{15-4\sqrt{194}}{65}$, and $\tan(p+q)=\frac{20+3\sqrt{194}}{15-4\sqrt{194}}$	
Candidate C - not considering $p+q$ is obtuse  $\cos(p+q)=\frac{33}{65}$				$\bullet^6$ ✗ $\bullet^7$ ✓$_1$	
4.	(c)		$\bullet^8$ find $\tan(p+q)$	$\bullet^8$ $-\frac{56}{33}$	**1**
Notes:					
8. Accept $-\frac{3640}{2145}$, $-\frac{728}{429}$ or $-\frac{280}{165}$ for $\bullet^8$.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)	(i)	•[1] interpret notation •[2] state expression for $f\left(g\left(x\right)\right)$	•[1] $f\left(x+3\right)$ **or** $2\left(g\left(x\right)\right)^2+5$ stated or implied by •[2] •[2] $2\left(x+3\right)^2+5$	**2**
		(ii)	•[3] state expression for $g\left(f\left(x\right)\right)$	•[3] $\left(2x^2+5\right)+3$	**1**
Notes:					
1. Accept $2g\left(x\right)^2+5$ for •[1].					
Commonly Observed Responses:					
Candidate A - two attempts $f\left(g\left(x\right)\right)=2x^2+5$ $=2\left(x+3\right)^2+5$ •[1] ✗ •[2] ✗				**Candidate B -** $f\left(g\left(x\right)\right)$ **and** $g\left(f\left(x\right)\right)$ **interchanged** $f\left(g\left(x\right)\right)=\left(2x^2+5\right)+3$ •[1] ✗ •[2] ✓[1] $g\left(f\left(x\right)\right)=2\left(x+3\right)^2+5$ •[3] ✓[1]	
	(b)		•[4] state range	•[4] $g\left(f\left(x\right)\right)\geq 8$	**1**
Notes:					
2. Accept $y\geq 8$ or $\left[8,\infty\right)$ for •[4].					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
6.			$\bullet^1$ state an appropriate pathway $\bullet^2$ express pathway in terms of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$	$\bullet^1$ eg $\overrightarrow{MH}+\overrightarrow{HE}+\overrightarrow{EN}$ stated **or** implied by $\bullet^2$ $\bullet^2$ $-\frac{1}{2}\mathbf{u}-\mathbf{v}+\frac{1}{3}\mathbf{w}$	**2**

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. Where an invalid pathway has been stated, award 0/2.
3. $\bullet^1$ may be awarded for the appearance of $\frac{1}{2}\mathbf{u}-\mathbf{u}-\mathbf{v}+\mathbf{w}-\frac{2}{3}\mathbf{w}$. $\bullet^2$ is not available for an unsimplified expression.
4. Where no pathway has been stated, see Candidates A and B.

Commonly Observed Responses:

Candidate A – no pathway stated and incorrect expression in $\mathbf{u}$, $\mathbf{v}$ AND $\mathbf{w}$ $-\frac{1}{2}\mathbf{u}-\mathbf{v}\ldots$ Award 1/2	**Candidate B – no pathway stated and incorrect expression in $\mathbf{v}$, $\mathbf{w}$ AND $\mathbf{u}$** $\ldots-\mathbf{v}+\frac{1}{3}\mathbf{w}$ Award 1/2

Question	Generic scheme	Illustrative scheme	Max mark
7.	$\bullet^1$ write second term in differentiable form $\bullet^2$ differentiate first term $\bullet^3$ differentiate second term $\bullet^4$ evaluate gradient	$\bullet^1$ $\ldots + 4x^{\frac{1}{2}}$ stated or implied by $\bullet^3$ $\bullet^2$ 2 $\bullet^3$ $\ldots + \frac{1}{2} \times 4x^{-\frac{1}{2}}$ $\bullet^4$ $\frac{8}{3}$ **or** $2\frac{2}{3}$	**4**

Notes:

1. $\bullet^3$ is only available for differentiating a term with a fractional index.
2. $\bullet^4$ is only available as a consequence of substituting into an expression involving a fractional power of x.
3. Where candidates make no attempt to differentiate, or integrate one or other of the terms, $\bullet^4$ is unavailable. See Candidates A, B and C.
4. $\bullet^4$ is not available where a candidate subsequently finds a perpendicular gradient.
5. Accept $y = \frac{8}{3}$ only where a substitution has been shown, no other values for y have been stated, or $\frac{8}{3}$ has been used as the gradient in the equation of the tangent.
6. Where candidates equate $\frac{dy}{dx}$ to 0, $\bullet^4$ is unavailable. See Candidate D.

Commonly Observed Responses:

Candidate A - limited evidence of differentiation of the second term

$y = 2x + 4x^{\frac{1}{2}}$ $\bullet^1$ ✓

$\frac{dy}{dx} = 2 + 2x^{\frac{3}{2}}$ $\bullet^2$ ✓ $\bullet^3$ ×

$2 + 2(9)^{\frac{3}{2}}$

$= 56$ $\bullet^4$ $✓_1$

Candidate B - second term integrated

$y = 2x + 4x^{\frac{1}{2}}$ $\bullet^1$ ✓

$\frac{dy}{dx} = 2 + \frac{8}{3}x^{\frac{3}{2}}$ $\bullet^2$ ✓ $\bullet^3$ × $\bullet^4$ ×

$2 + \frac{8}{3}(9)^{\frac{3}{2}}$

$= 74$

Candidate C - inclusion of $+ c$

$y = 2x + 4x^{\frac{1}{2}}$ $\bullet^1$ ✓

$\frac{dy}{dx} = 2 + 2x^{-\frac{1}{2}} + c$ $\bullet^2$ ✓ $\bullet^3$ × $\bullet^4$ ×

$m = \frac{8}{3}$

Candidate D - equating to 0

$y = 2x + 4x^{\frac{1}{2}}$ $\bullet^1$ ✓

$\frac{dy}{dx} = 2 + 2x^{-\frac{1}{2}} = 0$ $\bullet^2$ ✓ $\bullet^3$ ✓ $\bullet^4$ ×

$m = \frac{8}{3}$

Question			Generic scheme	Illustrative scheme	Max mark
8.			**Method 1** •[1] apply $\log_2 x+\log_2 y=\log_2 xy$ •[2] write in exponential form •[3] express in standard quadratic form •[4] solve quadratic and state solution of log equation **Method 2** •[1] apply $\log_2 x+\log_2 y=\log_2 xy$ •[2] apply $m\log_2 x=\log_2 x^m$ •[3] express in standard quadratic form •[4] solve quadratic and state solution of log equation	**Method 1** •[1] $\log_2\big(x\big(x+2\big)\big)=\ldots$ •[2] $x\big(x+2\big)=2^3$ •[3] $x^2+2x-8=0$ •[4] -4, 2 **and** $x>0\Rightarrow x=2$ **Method 2** •[1] $\log_2\big(x\big(x+2\big)\big)=\ldots$ •[2] $\ldots=\log_2 2^3$ •[3] $x^2+2x-8=0$ •[4] -4, 2 **and** $x>0\Rightarrow x=2$	**4**

Notes:

1. Accept $\log_2 x\big(x+2\big)=\ldots$ or $\log_2 x^2+2x=\ldots$ for •[1].
2. •[1] and •[2] may be awarded for $x\big(x+2\big)=2^3$. However, where 'log' has been scored out, see Candidates C, D and E.
3. Each line of working must be equivalent to the line above within a valid strategy.
4. •[2] is not available for $x\big(x+2\big)=3^2$. However, candidates may still gain •[3] and •[4]. See Candidate B.
5. •[3] and •[4] are only available if the quadratic reached at •[3] is obtained by applying the rules in •[1] and •[2]. See Candidates A and B.
6. •[4] is only available for solving a polynomial of degree two or higher.
7. At •[4], accept any indication that −4 has been discarded. For example, scoring out $x=-4$ or underlining $x=2$.
8. Do not penalise the omission of $x>0$ at •[4].

Question	Generic scheme	Illustrative scheme	Max mark
8.	(continued)		

Commonly Observed Responses:

Candidate A - $2^3 = 6$

$\log_2 x(x+2)=3$ $\bullet^1$ ✓

$x(x+2)=6$ $\bullet^2$ ×

$x^2+2x-6=0$ $\bullet^3$ ✓1

$x=-1\pm\sqrt{7}$

$x>0 \Rightarrow x=\sqrt{7}-1$ $\bullet^4$ ✓1

Candidate B

$\log_2 x(x+2)=3$ $\bullet^1$ ✓

$x(x+2)=9$ $\bullet^2$ ×

$x^2+2x-9=0$ $\bullet^3$ ✓1

$x=-1\pm\sqrt{10}$

$x>0 \Rightarrow x=\sqrt{10}-1$ $\bullet^4$ ✓1

Candidate C - valid with and without scoring out

~~$\log_2$~~ $x(x+2)=$ ~~$\log_2$~~ 8 $\bullet^1$ ✓

$x(x+2)=8$ $\bullet^2$ ✓

$x^2+2x-8=0$ $\bullet^3$ ✓

$x=$ ~~-4~~, 2 $\bullet^4$ ✓

$\bullet^3$ and $\bullet^4$ are available

Candidate D - invalid with and without scoring out

~~$\log_2$~~ $x(x+2)=$ ~~$\log_2$~~ 3 $\bullet^1$ ✓ - BOD

$x(x+2)=8$ $\bullet^2$ ×

$x^2+2x-8=0$ $\bullet^3$ ×

$x=$ ~~-4~~, 2 $\bullet^4$ ×

$\bullet^3$ and $\bullet^4$ are **not** available

Candidate E - invalid with and without scoring out

~~$\log_2$~~ $x(x+2)=$ ~~$\log_2$~~ 3 $\bullet^1$ ✓ - BOD

$x(x+2)=3$ $\bullet^2$ ×

$x^2+2x-3=0$ $\bullet^3$ ×

$x=$ ~~-3~~, 1 $\bullet^4$ ×

$\bullet^3$ and $\bullet^4$ are **not** available

Candidate F - invalid processing of logs

$\log_2 x(x+2)=2^3$ $\bullet^1$ ✓ $\bullet^2$ ×

$x(x+2)=8$

$x^2+2x-8=0$ $\bullet^3$ ×

$x=$ ~~-4~~, 2 $\bullet^4$ ×

$\bullet^3$ and $\bullet^4$ are **not** available

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		$\bullet^1$ find an appropriate vector eg $\overrightarrow{DE}$ $\bullet^2$ find a second vector eg $\overrightarrow{EF}$ **and** compare $\bullet^3$ appropriate conclusion	$\bullet^1$ eg $\overrightarrow{DE} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ $\bullet^2$ eg $\overrightarrow{EF} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \therefore \overrightarrow{DE} = 2\overrightarrow{EF}$ $\bullet^3$ $\Rightarrow$DE is parallel to EF (common direction) and E is a common point $\Rightarrow$ D,E and F are collinear	**3**

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. The comparison at $\bullet^2$ may be expressed as a ratio, for example DE : EF = 2 : 1.
3. If no comparison of vectors or the trivial comparison '$\overrightarrow{DE} = \overrightarrow{EF}$' is made at $\bullet^2$, then $\bullet^3$ is not available.
4. $\bullet^3$ can only be awarded if a candidate has stated 'parallel', 'common point' and 'collinear'.
5. Candidates who state or imply that 'points are parallel' or 'vectors are collinear' or state 'parallel and share a common point $\Rightarrow$collinear' do not gain $\bullet^3$. There must be reference to the points.
6. Do not accept 'd, e and f are collinear' at $\bullet^3$.

Commonly Observed Responses:

Candidate A - missing labels

$\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ $\bullet^1$ ^

$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \therefore \overrightarrow{DE} = 2\overrightarrow{EF}$ $\bullet^2$ ✓$_1$

Missing labels treated as a repeated error

$\Rightarrow$ DE is parallel to EF (common direction)
and E is a common point
$\Rightarrow$ D,E and F are collinear $\bullet^3$ ✓$_1$

Candidate B

$\overrightarrow{EF} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\bullet^1$ ✓

$\overrightarrow{DE} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\bullet^2$ ✓

$\therefore 2\overrightarrow{DE} = \overrightarrow{EF}$

ignore working subsequent to correct statement on previous line

$\Rightarrow$ DE is parallel to EF (common direction)
and E is a common point
$\Rightarrow$ D,E and F are collinear $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		**Method 1** •4 interpret ratio •5 find coordinates of G **Method 2** •4 interpret ratio •5 find coordinates of G **Method 3** •4 use section formula •5 find coordinates of G	**Method 1** •4 $\begin{pmatrix}3\\-6\\9\end{pmatrix}$ **and** $\begin{pmatrix}4\\-8\\12\end{pmatrix}$ **or** $\begin{pmatrix}7\\-14\\21\end{pmatrix}$ •5 (6,−8,22) **Method 2** •4 eg $\overrightarrow{FG}=\frac{4}{3}\overrightarrow{DF}$ **or** $\overrightarrow{DG}=\frac{7}{3}\overrightarrow{DF}$ •5 (6,−8,22) **Method 3** •4 $\frac{1}{3}(-4\mathbf{d}+7\mathbf{f})$ **or** $\mathbf{f}=\frac{3}{7}\mathbf{g}+\frac{4}{7}\mathbf{d}$ •5 (6,−8,22)	**2**

Notes:

7. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).

8. For (6,−8,22) without working award 2/2.

9. For $\begin{pmatrix}6\\-8\\22\end{pmatrix}$ without working award 1/2.

10. For $\left(\frac{17}{4},-\frac{9}{2},\frac{67}{4}\right)$ (ratio of 4:3 with working) award 1/2. See Candidate C.

11. No marks are available for finding an internal division point, for example $\left(\frac{2}{7},\frac{24}{7},\frac{34}{7}\right)$ or using other ratios.

Question		Generic scheme	Illustrative scheme	Max mark
9.	(b)	(continued)		

Commonly Observed Responses:

Candidate C - ratio of 4:3

$\overrightarrow{FG} = \frac{3}{4}\overrightarrow{DF}$ •4 ✗

$G = \left(\frac{17}{4}, -\frac{9}{2}, \frac{67}{4}\right)$ •5 ✓1

Candidate D

$\frac{\overrightarrow{FG}}{\overrightarrow{DF}} = \frac{4}{3}$ •4 ✓

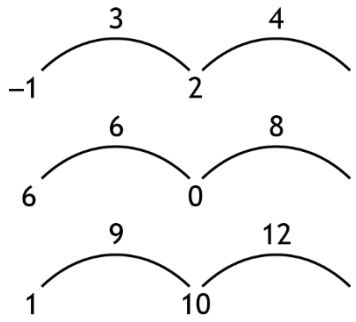
$3\overrightarrow{FG} = 4\overrightarrow{DF}$

$3(\mathbf{g} - \mathbf{f}) = 4(\mathbf{f} - \mathbf{d})$

$3\mathbf{g} = 7\mathbf{f} - 4\mathbf{d}$

G = (6,−8,22) •5 ✓

Candidate E - stepping out using absolute values



•4 ✓

G = (6,−8,22) •5 ✓

Candidate F - Component form of section formula

$\frac{4(-1)+3x}{7} = 2$

$\frac{4(6)+3y}{7} = 0$

$\frac{4(1)+3z}{7} = 10$ •4 ✓

G = (6,−8,22) •5 ✓

Question	Generic scheme	Illustrative scheme	Max mark
10.	$\bullet^1$ start to integrate	$\bullet^1$ $-6\cos\left(3x-\frac{\pi}{2}\right)$	**4**
	$\bullet^2$ complete integration	$\bullet^2$ $\cdots\times\frac{1}{3}$	
	$\bullet^3$ substitute limits	$\bullet^3$ $\left(-2\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)$ $-\left(-2\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$	
	$\bullet^4$ evaluate integral	$\bullet^4$ 2	

Notes:

1. Award $\bullet^1$ for the appearance of $-6\cos\left(3x-\frac{\pi}{2}\right)$ regardless of any **constant** multiplier.
2. For candidates who differentiate throughout, make no attempt to integrate, or use another invalid approach – for example $\cos\left(3x-\frac{\pi}{2}\right)^2$ or $6\int\sin 3x-\sin\frac{\pi}{2}dx$ – award 0/4.
3. For candidates who write $6-\cos(\ldots$, no further marks are available unless the lack of brackets is corrected on the next line of working.
4. Do not penalise the inclusion of '$+c$' at $\bullet^2$ or $\bullet^3$.
5. Do not penalise the continued appearance of the integral sign or 'dx' after integrating.
6. Accept $\left(-2\cos\left(\frac{\pi}{2}\right)\right)-\left(-2\cos(0)\right)$ or $\left(-2\cos(3\times60-90)\right)-\left(-2\cos(3\times30-90)\right)$ for $\bullet^3$.
7. $\bullet^4$ is only available where candidates have considered both limits within a trigonometric function.
8. Where candidates use a mixture of degrees and radians, for example, $\left(-2\cos\left(3\times60°-\frac{\pi}{2}\right)\right)$ $-\left(-2\cos\left(3\times30°-\frac{\pi}{2}\right)\right)$, $\bullet^3$ is not awarded. However, $\bullet^4$ is still available.

Commonly Observed Responses:

Candidate A - incomplete integration

$-6\cos\left(3x-\frac{\pi}{2}\right)$ $\bullet^1$ ✓ $\bullet^2$ ^

$\left(-6\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)-\left(-6\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$ $\bullet^3$ ✓1

6 $\bullet^4$ ✓1

Candidate B - integrating over two lines

$\int 6\sin\left(3x-\frac{\pi}{2}\right)dx$

$=-6\cos\left(3x-\frac{\pi}{2}\right)$ $\bullet^1$ ✓

$=-6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ $\bullet^2$ ✗

$\bullet^3$ and $\bullet^4$ are available

Question	Generic scheme	Illustrative scheme	Max mark
10.	(continued)		
Commonly Observed Responses (continued):			
Candidate C - integrated in part $6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ $\bullet^1$ ✗ $\bullet^2$ ✓$_1$ $\bullet^3$ and $\bullet^4$ are available		**Candidate D - integrated in part** $-6\cos\left(3x-\frac{\pi}{2}\right)\times 3$ $\bullet^1$ ✓ $\bullet^2$ ✗ $\bullet^3$ and $\bullet^4$ are available	
Candidate E - insufficient evidence of integration $6\cos\left(3x-\frac{\pi}{2}\right)$ $\bullet^1$ ✗ $\bullet^2$ ^ $\bullet^3$ and $\bullet^4$ are **not** available		**Candidate F - invalid approach** $\pm 6\sin\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$ and $\bullet^4$ are **not** available	
Candidate G - working in degrees before integrating $\int_{30}^{60} 6\sin(3x-90)\,dx$ $-6\cos(3x-90)\times\frac{1}{3}$ $\bullet^1$ ✗ $\bullet^2$ ✗ $(-2\cos(90))-(-2\cos(0))$ $\bullet^3$ ✓$_1$ 2 $\bullet^4$ ✓$_1$		**Candidate H - BEWARE** $-6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\left(-2\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)-\left(-2\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$ $\bullet^3$ ✓ $\left(-2\cos\left(\frac{\pi}{2}\right)\right)-(-2\cos\ \ )$ $\bullet^4$ ✗ 2	

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(i)	**Method 1** •[1] know to use $x=-2$ in synthetic division •[2] complete division, interpret result and state conclusion **Method 2** •[1] know to substitute $x=-2$ •[2] complete evaluation, interpret result and state conclusion	**Method 1** •[1] -2 | 1 7 18 16 1 •[2] -2 | 1 7 18 16 -2 -10 -16 1 5 8 0 Remainder $=0\therefore(x+2)$ is a factor **Method 2** •[1] $(-2)^3+7(-2)^2+18(-2)+16$ •[2] $=0\therefore(x+2)$ is a factor	2

Notes:

1. Communication at •[2] must be consistent with working at that stage i.e. candidate's working must arrive legitimately at 0 before •[2] can be awarded.
2. The conclusion for (a)(i) must be stated in (a)(i).
3. Accept any of the following for •[2]:
   - '$f(-2)=0$ so $(x+2)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', ∴, →, ⇒ etc
4. Do not accept any of the following for •[2]:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-2$ is a factor', '… is a root'
   - the word 'factor' only, with no link

Commonly Observed Responses:

Candidate A - grid method

	x^2	$5x$	8
x	x^3	$5x^2$	
2	$2x^2$		

•[1] ✓

	x^2	$5x$	8
x	x^3	$5x^2$	$8x$
2	$2x^2$	$10x$	16

with no remainder

$\therefore(x+2)$ is a factor •[2] ✓

Candidate B - grid method

	x^2	$5x$	8
x	x^3	$5x^2$	
2	$2x^2$		

•[1] ✓

	x^2	$5x$	8
x	x^3	$5x^2$	$8x$
2	$2x^2$	$10x$	16

$\therefore(x+2)(x^2+5x+8)=x^3+7x+18x+16$

$\therefore(x+2)$ is a factor •[2] ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(ii)	$\bullet^3$ use discriminant $\bullet^4$ evaluate discriminant and give valid reason	$\bullet^3$ $5^2-4(1)(8)$ $\bullet^4$ $-7<0$, $\Rightarrow x^2+5x+8$ does not factorise (so $(x+2)$ is the only linear factor)	2

Notes:

5. Evidence for $\bullet^3$ may appear in the quadratic formula.
6. Accept $b^2-4ac=-7$ for $\bullet^3$.
7. Do not penalise candidates who replace "x^2+5x+8" with "the quadratic factor".
8. At $\bullet^4$ accept interpretations such as "no real roots", "no (further) factors" and "cannot factorise (further)" with comparison with 0.
9. Where the quadratic factor used in $\bullet^3$ can be factorised, $\bullet^4$ is not available.

Commonly Observed Responses:

Candidate C - no reference to discriminant

$-7<0$ $\bullet^3$ ^

$\Rightarrow$ the quadratic factor does not factorise $\bullet^4$ ^

$\Rightarrow (x+2)$ is the only linear factor

Candidate D - minimal reference to discriminant

$-7<0$ $\bullet^3$ ^

discriminant < 0

$\Rightarrow$ the quadratic factor does not factorise $\bullet^4$ ✓

$\Rightarrow (x+2)$ is the only linear factor

Candidate E - no evaluation of discriminant

$b^2-4ac<0$ $\bullet^3$ ^ $\bullet^4$ ^

$\Rightarrow$ the quadratic factor does not factorise

$\Rightarrow (x+2)$ is the only linear factor

Candidate F - no evaluation of discriminant

Product = 8 and Sum = 5

No integer pair have this product and sum so the quadratic factor does not factorise $\bullet^3$ ^ $\bullet^4$ ^

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		$\bullet^5$ equate curves	$\bullet^5$ $2x^3+20x^2+27x+9=6x^2-9x-23$	4
			$\bullet^6$ rearrange and link to (a)	$\bullet^6$ $2x^3+14x^2+36x+32=0$ leading to $2\left(x^3+7x^2+18x+16\right)=0$	
			$\bullet^7$ find x-coordinate	$\bullet^7$ $x=-2$	
			$\bullet^8$ find y-coordinate	$\bullet^8$ $y=19$	

Notes:

10. At $\bullet^6$ do not penalise candidates who have divided the cubic equation by 2.
11. '$=0$' must appear in at least one of the equations at the $\bullet^6$ stage.
12. $\bullet^7$ may not be awarded for working in part (a).
13. Where an error has occurred at the $\bullet^5$ or $\bullet^6$ stage, $\bullet^7$ and $\bullet^8$ are still available for finding **all** the solutions (the discriminant need not be used for $\bullet^8$ to be awarded on follow-through).
14. Where candidates "factorise" an irreducible quadratic or use another invalid strategy, $\bullet^7$ and $\bullet^8$ are not available.

Commonly Observed Responses:

Candidate G - not explicitly using part (a)

$2x^3+20x^2+27x+9=6x^2-9x-23$ $\bullet^5$ ✓

$2x^3+14x^2+36x+32=0$

-2	2	14	36	32
		-4	-20	-32
	2	10	16	0

$(x+2)(2x^2+10x+16)=0$ $\bullet^6$ ✓

$2(x+2)(x^2+5x+8)=0$

To gain $\bullet^7$, x^2+5x+8 must also be stated in (a) or the discriminant used.
Also accept $x=-2$ or $x+2=0$,
AND $x^2+5x+8=0$ for $\bullet^6$
(or without $=0$ if previously included)

$x=-2$ $\bullet^7$ ✓

$y=19$ $\bullet^8$ ✓

Candidate H - not using part (a)

$2x^3+20x^2+27x+9=6x^2-9x-23$ $\bullet^5$ ✓

$2x^3+14x^2+36x+32=0$

-2	2	14	36	32
		-4	-20	-32
	2	10	16	0

$(x+2)(2x^2+10x+16)$ $\bullet^6$ ✓

Also accept $x=-2$ or $x+2=0$,
AND $2x^2+10x+16=0$ for $\bullet^6$
(or without $=0$ if previously included)

$x=-2$ and $10^2-4(2)(16)=-28<0$ so

$x=-2$ is the only root $\bullet^7$ ✓

$y=19$ $\bullet^8$ ✓

Question		Generic scheme	Illustrative scheme	Max mark
11.	(b)	**(continued)**		

Commonly Observed Responses (continued):

Candidate I - no evidence of link to part (a)

$2x^3+20x^2+27x+9=6x^2-9x-23$ •5 ✓

$2x^3+14x^2+36x+32=0$ •6 ^ •7 ^

$x=-2$

$y=19$ •8 ✓1

Candidate J - subtracting equations

$0=\left(2x^3+20x^2+27x+9\right)-\left(6x^2-9x-23\right)$ •5 ✓

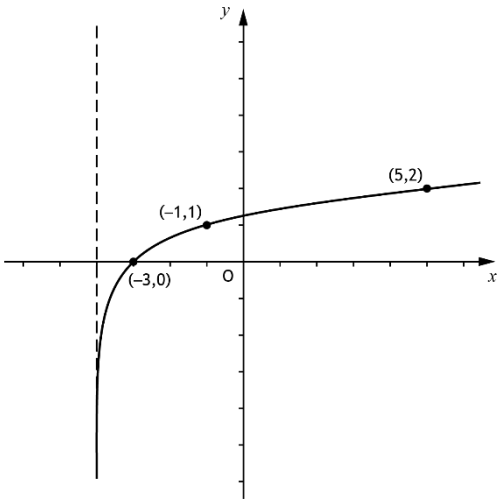
$0=2x^3+14x^2+36x+32$

$0=2\left(x^3+7x^2+18x+16\right)$ •6 ✓

However...

$y=\left(2x^3+20x^2+27x+9\right)-\left(6x^2-9x-23\right)$ •5 ✗

$0=2\left(x^3+7x^2+18x+16\right)$ •6 ✓1

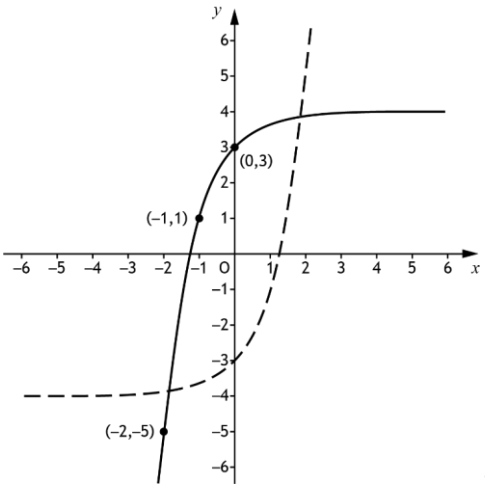
Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] graph reflected in $y = x$ i.e. generally increasing for $-3 \le x \le 5$ and passing through (−3,0), (−1,1) and (5,2) •[2] increasing for $x < -3$ (and for $x > 5$) and approaching $x = -4$ from the right	•[1], •[2]  	**2**

Notes:

1. •[1] is not available where the graph has a maximum turning point where $-1 < x < 5$ and •[2] is not available where the graph has a maximum turning point where $5 \le x \le 6$.
2. •[2] is only available where •[1] has been awarded.
3. The line $x = -4$ does not need to be shown.
4. For a rotation, award 0/2 – for example see Candidate A.
5. Do not penalise graphs which do not extend beyond (5,2).

Commonly Observed Responses:

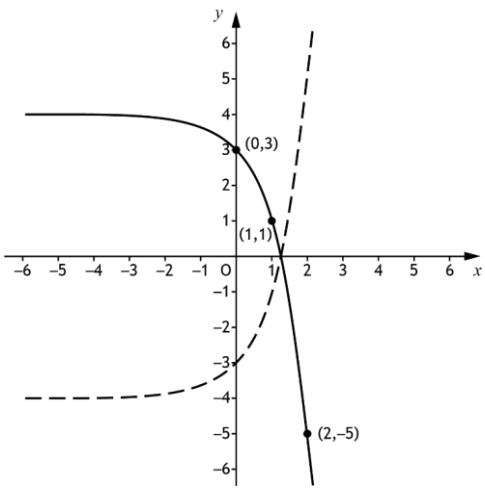
Candidate A - rotation about O



•[1] × •[2] ×

No further marks in this question can be inferred *from this graph*.
In part (b), candidates may work with the original graph and gain full marks for $b = 4$ and $a = 3$ (with or without working) and $x > -4$.

Candidate B - reflection in x-axis



•[1] × •[2] ×

No further marks in this question can be inferred *from this graph*.
In part (b), candidates may work with the original graph and gain full marks for $b = 4$ and $a = 3$ (with or without working) and $x > -4$.

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)	(i)	$\bullet^3$ determine value of b $\bullet^4$ determine value of a	$\bullet^3$ $b=4$ $\bullet^4$ $a=3$	**2**
		(ii)	$\bullet^5$ state domain	$\bullet^5$ $x>-4, \quad (x\in\mathbb{R})$	**1**

Notes:

6. a and b may be calculated independently of each other.
7. The values of a or b can be calculated using $a=b-1$ where $b>1$.
8. Accept '$f(x)>-4$' for $\bullet^5$.
9. For $\bullet^5$, the domain can be calculated using $x>-b$.
10. For $\bullet^5$, do not penalise the omission of $x\in\mathbb{R}$.
11. For $\bullet^5$, the domain can be from the original graph, the candidate's transformed graph or from the value of b in (b)(i). However, refer back to Candidates A and B.

Commonly Observed Responses:

Candidate C - Values of a and b in equation

$\log_3(x+4)$ $\bullet^3$ ✓ $\bullet^4$ ✓

Candidate D - Incorrect values of a and b

$\log_3(x+4)$ $\bullet^3$ ×

$b=3$ $\bullet^4$ ×

$x>-3$ $\bullet^5$ $\checkmark_1$

[END OF MARKING INSTRUCTIONS]

National
Qualifications
2026

2026 Mathematics

Higher – Paper 2

Question Paper Finalised Marking Instructions

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General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ $\quad$ $\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$ $\quad$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, not a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\quad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\quad$ $\frac{4\!\!\not/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)(2x+1)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•1 use discriminant •2 apply condition and express in standard quadratic form •3 state values of k	•1 $k^2-4(1)(k+3)$ •2 $k^2-4k-12=0$ •3 −2, 6	3

Notes:

1. Where candidates state an incorrect or no condition, •2 is not available. However, •3 is available for finding the roots of the quadratic - see Candidate B.
2. Where x appears in any expression, no further marks are available.

Commonly Observed Responses:

Candidate A

(For equal roots) $b^2-4ac=0$

$k^2-4(1)(k+3)$ •1 ✓

$k^2-4k-12$ •2 ✓

$k=-2,6$ •3 ✓

Candidate B

(For equal roots) $b^2-4ac>0$ •2 ×

$k^2-4(1)(k+3)$ •1 ✓

$k^2-4k-12$

$k=-2,6$ •3 ✓1

⋮

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		$\bullet^1$ evaluate $\mathbf{u}\cdot\mathbf{v}$	$\bullet^1$ 5	**1**
Notes:					
Commonly Observed Responses:					
Candidate A - poor notation $\begin{pmatrix}3\\2\\6\end{pmatrix}\cdot\begin{pmatrix}-5\\-2\\4\end{pmatrix}=\begin{pmatrix}-15\\-4\\24\end{pmatrix}=5$ $\bullet^1$ ×					
	(b)		$\bullet^2$ evaluate $\lvert\mathbf{u}\rvert$ $\bullet^3$ evaluate $\lvert\mathbf{v}\rvert$ $\bullet^4$ substitute into formula for scalar product $\bullet^5$ calculate angle	$\bullet^2$ $\lvert\mathbf{u}\rvert=7$ or $\sqrt{49}$ $\bullet^3$ $\lvert\mathbf{v}\rvert=\sqrt{45}$ $\bullet^4$ $\cos\theta=\dfrac{5}{7\sqrt{45}}$ $\bullet^5$ 83.887° **OR** 1.464 radians	**4**
Notes:					

1. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. For example, accept $\sqrt{5^2+2^2+4^2}=\sqrt{45}$ or $\sqrt{-5^2+-2^2+4^2}=\sqrt{45}$ for $\bullet^3$. However, do not accept $\sqrt{5^2-2^2+4^2}=\sqrt{45}$ for $\bullet^3$.
2. $\bullet^4$ is not available to candidates who simply state the formula $\cos\theta=\dfrac{\mathbf{u}.\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$. However, $7\times\sqrt{45}\times\cos\theta=5$ or $\theta=\cos^{-1}\left(\dfrac{5}{7\sqrt{45}}\right)$ is acceptable for $\bullet^4$.
3. Accept correct answers rounded to 84° or 1.5 radians (or 93 gradians).
4. Do not penalise the omission or incorrect use of units.
5. $\bullet^5$ is only available as a result of using a valid strategy.
6. $\bullet^5$ is only available for a single **acute** angle.
7. For a correct answer with no working award 0/4.

Question	Generic scheme	Illustrative scheme	Max mark
2. (b) (continued)			
Commonly Observed Responses:			
Candidate B - minimal communication $\cos^{-1}\left(\frac{5}{\sqrt{49}\sqrt{45}}\right)$ $\bullet^2$ ✓ $\bullet^3$ ✓ $\bullet^4$ ✓ $=83.9°$ $\bullet^5$ ✓		**Candidate C - minimal communication** $\cos^{-1}\left(\frac{5}{\sqrt{3^2+2^2+6^2}\times\sqrt{(-5)^2+(-2)^2+4^2}}\right)$ $\bullet^4$ ✓ $=83.9°$ $\bullet^2$ ✓ $\bullet^3$ ✓ $\bullet^5$ ✓	
Candidate D - insufficient communication $\bullet^2$ $\lvert\mathbf{u}\rvert=7$ $\bullet^2$ ✓ $\bullet^3$ $\lvert\mathbf{v}\rvert=\sqrt{45}$ $\bullet^3$ ✓ $\bullet^4$ $\frac{5}{7\sqrt{45}}$ $\bullet^4$ ^ $\bullet^5$ 83.887° **OR** 1.464 radians $\bullet^5$ $✓_1$			

Question	Generic scheme	Illustrative scheme	Max mark
3.	**Method 1**	**Method 1**	**3**
	•1 equate composite function to x	•1 $g\left(g^{-1}(x)\right)=x$	
	•2 write $g\left(g^{-1}(x)\right)$ in terms of $g^{-1}(x)$	•2 $\frac{1}{5}g^{-1}(x)+6=x$	
	•3 state inverse function	•3 $g^{-1}(x)=5(x-6)$	
	Method 2	**Method 2**	
	•1 write as $y=g(x)$ and start to re-arrange	•1 $y=g(x)\Rightarrow x=g^{-1}(y)$ $y-6=\frac{1}{5}x$	
	•2 express x in terms of y	•2 eg $x=\frac{(y-6)}{\frac{1}{5}}$	
	•3 state inverse function	•3 $g^{-1}(y)=5(y-6)$ $\Rightarrow g^{-1}(x)=5(x-6)$	

Notes:

1. In Method 1, accept $\frac{1}{5}g^{-1}(x)+6=x$ for •1 and •2.
2. In Method 2 accept $y-6=\frac{1}{5}x$ without reference to $y=g(x)\Rightarrow x=g^{-1}(y)$ at •1.
3. In Method 2, accept $g^{-1}(x)=5(x-6)$ without reference to $g^{-1}(y)$ at •3.
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent.
5. At •3 stage, accept g^{-1} written in terms of any dummy variable eg $g^{-1}(y)=5(y-6)$.
6. $y=5(x-6)$ does not gain •3.
7. $g^{-1}(x)=5(x-6)$ with no working gains 3/3.
8. In method 2, where candidates make multiple algebraic errors at the •1 or •2 stage, •3 is still available. For example, see Candidate G.
9. Marks should only be awarded for using a valid strategy to find the inverse of $g(x)$.

Question	Generic scheme	Illustrative scheme	Max mark

3. (continued)

Commonly Observed Responses:

Candidate A

$g(x)=\frac{1}{5}x+6$

$y=\frac{1}{5}x+6$

$x=5(y-6)$ $\bullet^1$ ✓ $\bullet^2$ ✓

$y=5(x-6)$ $\bullet^3$ ×

$g^{-1}(x)=5(x-6)$

Candidate B

$g(x)=\frac{1}{5}x+6$

$y=\frac{1}{5}x+6$

$x=\frac{1}{5}y+6$ $\bullet^1$ ×

$y=5(x-6)$ $\bullet^2$ ✓$_1$

$g^{-1}(x)=5(x-6)$ $\bullet^3$ ✓$_1$

Candidate C

$g(x)=\frac{1}{5}x+6$

$x=\frac{1}{5}g(x)+6$ $\bullet^1$ ×

$g(x)=5(x-6)$ $\bullet^2$ ✓$_1$

$g^{-1}(x)=5(x-6)$ $\bullet^3$ ✓$_1$

The first line appears in the question

Candidate D - Method 1

$g\left(g^{-1}(x)\right)=\frac{1}{5}g^{-1}(x)+6$ $\bullet^2$ ✓

$x=\frac{1}{5}g^{-1}(x)+6$ $\bullet^1$ ✓

$g^{-1}(x)=5(x-6)$ $\bullet^3$ ✓

Candidate E - BEWARE

$g'(x)=\ldots$ $\bullet^3$ ×

OR

$f^{-1}(x)=\ldots$ $\bullet^3$ ×

OR

multiple expressions for $g^{-1}(x)$ $\bullet^3$ ×

Candidate F

$x\rightarrow x\div 5\rightarrow x\div 5+6=g(x)$

$\div 5\rightarrow +6$

$\therefore -6\rightarrow \times 5$ $\bullet^1$ ✓

$5(x-6)$ $\bullet^2$ ✓

$g^{-1}(x)=5(x-6)$ $\bullet^3$ ✓

Candidate G - incomplete multiplication

$y=\frac{1}{5}x+6$

$5y=x+6$ $\bullet^1$ ×

$x=5y-6$ $\bullet^2$ ✓$_1$

$g^{-1}(x)=5x-6$ $\bullet^3$ ✓$_1$

Candidate H - poor notation

$y=\frac{1}{5x}+6$

$y-6=\frac{1}{5x}$

$x=5(y-6)$

$g^{-1}(x)=5(x-6)$ Award 2/3

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 determine distance between points •2 find equation of circle	•1 $\sqrt{29}$ stated or implied by •2 •2 $(x+2)^2+(y-5)^2=29$	2

Notes:

1. Accept $r^2=29$ **or** $d^2=29$ for •1.
2. •2 is only available where candidates have used a valid strategy to find the radius.
3. •2 is not available to candidates who have a negative value for r or for r^2.

Commonly Observed Responses:

Candidate A

$(x-3)^2+(y-7)^2=29$ •1 ✓ •2 ✗

Candidate B - unsimplified answers

$(x+2)^2+(y-5)^2=\sqrt{29}^2$ •1 ✓ •2 ^

$(x-(-2))^2+(y-5)^2=29$ •1 ✓ •2 ^

Candidate C - general form

$g=+2$, $f=-5$

$x^2+y^2+2gx+2fy+c=0$

$3^2+7^2+2(2)(3)+2(-5)(7)+c=0$

$c=0$ •1 ✓

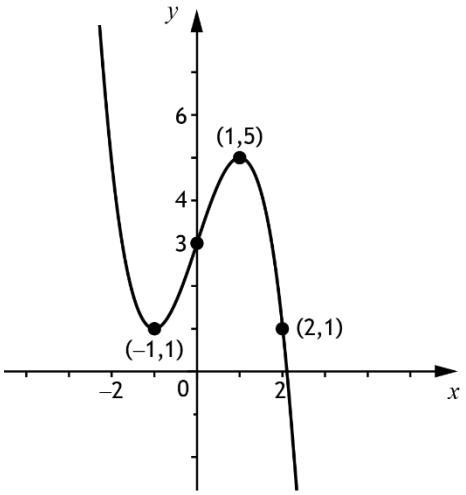
$x^2+y^2+4x-10y=0$ •2 ✓

Candidate D - BEWARE

$\sqrt{3^2-(-2)^2+7^2-5^2}=\sqrt{29}$ •1 ✗ •2 ✗

$\sqrt{(\pm2)^2+(\pm5)^2}=\sqrt{29}$ •1 ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ find value of u_1	$\bullet^1$ 7.25 or $\frac{29}{4}$	**1**
Notes:					
Commonly Observed Responses:					
	(b)		$\bullet^2$ communicate condition	$\bullet^2$ $1.125 > 1$	**1**
Notes:					
1. For $\bullet^2$, accept statements such as: • '1.125 is greater than 1' • '1.125 does not fall between −1 and 1' • '$-1 < 1.125 < 1$ is false' • '1.125 is not in $-1 < a < 1$'. 2. $\bullet^2$ is not available for statements such as: • '$1.125 \geq 1$' • 'It is greater than 1' • '1.125 does not fall between 0 and 1' • '$-1 < 1.125 \not< 1$' • '$-1 < 1.125 > 1$'. 3. Candidates who state '$a > 1$' can only gain $\bullet^2$ if it is explicitly stated that $a = 1.125$.					
Commonly Observed Responses:					
	(c)		$\bullet^3$ find value of u_4 $\bullet^4$ find value of u_5 and state value of n	$\bullet^3$ $u_4 = 27.275\ldots$ $\bullet^4$ $u_5 = 35.685\ldots$, **AND** $n = 5$	**2**
Notes:					
4. The value of u_4 may appear within a substitution to find u_5. 5. Do not penalise truncated or rounded values. 6. For $n = 5$ with no working, award 0/2. 7. Where $n = 5$ has not been explicitly stated, $\bullet^4$ may be awarded where u_5 has been explicitly compared to 30. See Candidate A.					
Commonly Observed Responses:					
Candidate A - not stating value of *n* $u_5 = 35.685$ AND $u_5 > 30$ $\bullet^4$ ✓ $u_5 = 35.685 > 30$ $\bullet^4$ ✓ $\underline{\underline{u_5}} = 35.685$ $\bullet^4$ ^					

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] vertical reflection in x-axis identifiable from stationary points of graph •[2] vertical translation of '+1' units identifiable from stationary points of graph •[3] transformation applied in the correct order and remaining points identifiable from graph	•[1,2,3] 	**3**

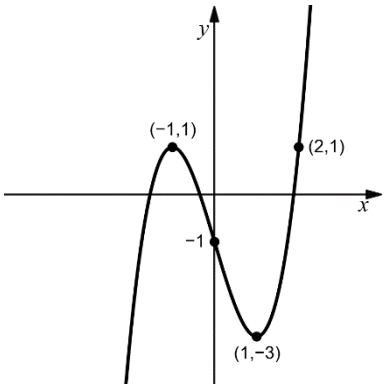
Notes:

1. Where candidates do not sketch a cubic function of the correct orientation, award 0/3. However, see Candidate A.
2. Where either stationary point has been incorrectly transformed, no marks are available. However, see Candidates A and B and the table for possible exceptions.
3. Do not penalise graphs which cross the x-axis at (2,0) or which do not extend past (2,1).

Commonly Observed Responses:

Candidate A – one transformation

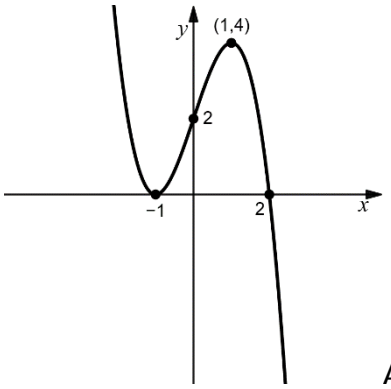
$y = f(x) + 1$



Award 1/3

Candidate B – one transformation

$y = -f(x)$



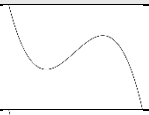
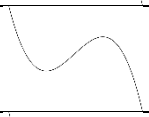
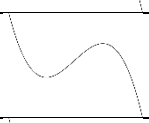
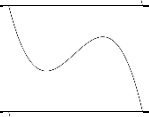
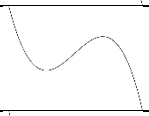
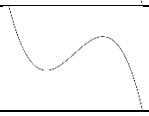
Award 1/3

Question	Generic scheme	Illustrative scheme	Max mark

6. (continued)

Commonly Observed Responses (continued):

Table showing two transformations…

Function	Transformation of stationary points	Transformation of intercepts	Shape	Award
$y=-f(x)-1$	(−1,−1) and (1,3)	(2,−1) and (0,1)		**2/3**
$y=f(-x)+1$	(1,1) and (−1,−3)	(−2,1) and (0,−1)		**1/3**
$y=-f(x+1)$	(−2,0) and (0,4)	(1,0) and (−1,2)		**1/3**
$y=-f(x-1)$	(0,0) and (2,4)	(3,0) and (1,2)		**1/3**
$y=f((-x)+1)$	(2,0) and (0,−4)	(−1,0) and (1,−2)		**0/3**
$y=f((-x)-1)$	(0,0) and (−2,−4)	(−3,0) and (−1,−2)		**0/3**

Question	Generic scheme	Illustrative scheme	Max mark
7.	**Method 1**	**Method 1**	**4**
	$\bullet^1$ state integral which represents the area	$\bullet^1$ eg $-\int_{-1}^{2}\left(x^2-x-2\right)dx$	
		OR	
		$\int_{-1}^{2}\left(-x^2+x+2\right)dx$	
	$\bullet^2$ integrate valid expression	$\bullet^2$ eg $-\frac{x^3}{3}+\frac{x^2}{2}+2x$	
	$\bullet^3$ substitute limits	$\bullet^3$ $\left(-\frac{(2)^3}{3}+\frac{(2)^2}{2}+2(2)\right)$ $-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2(-1)\right)$	
	$\bullet^4$ evaluate area	$\bullet^4$ $\frac{9}{2}\left(\text{units}^2\right)$	
	Method 2	**Method 2**	
	$\bullet^1$ state integral which is related to the area	$\bullet^1$ eg $\int_{-1}^{2}\left(x^2-x-2\right)dx$	
	$\bullet^2$ integrate valid expression	$\bullet^2$ eg $\frac{x^3}{3}-\frac{x^2}{2}-2x$	
	$\bullet^3$ substitute limits	$\bullet^3$ $\left(\frac{2^3}{3}-\frac{2^2}{2}-2(2)\right)$ $-\left(\frac{(-1)^3}{3}-\frac{(-1)^2}{2}-2(-1)\right)$	
	$\bullet^4$ evaluate integral and state area	$\bullet^4$ $-\frac{9}{2}\therefore$ area $=\frac{9}{2}\left(\text{units}^2\right)$	

Notes:

1. $\bullet^1$ is not available to candidates who omit 'dx'.
2. Limits must appear at the $\bullet^1$ stage for $\bullet^1$ to be awarded.
3. In Method 1, accept $\int_{-1}^{2}(0)-\left(x^2-x-2\right)dx$ **OR** $\int_{2}^{-1}\left(x^2-x-2\right)dx$ for $\bullet^1$.
4. Where candidates differentiate one or more terms at $\bullet^2$, then $\bullet^3$ and $\bullet^4$ are unavailable.
5. Candidates who substitute limits without integrating do not gain $\bullet^3$ and $\bullet^4$.
6. Do not penalise the inclusion of '$+c$' at $\bullet^2$ and $\bullet^3$.
7. Do not penalise the continued appearance of the integral sign or 'dx' after $\bullet^1$.
8. In Method 2, where a candidate's integral leads to a positive value, $\bullet^4$ is not available.

Question	Generic scheme	Illustrative scheme	Max mark

7. (continued)

Commonly Observed Responses:

Candidate A - missing working

$-\int_{-1}^{2}\left(x^2-x-2\right)$ $\bullet^1$ ^

$=-\frac{x^3}{3}+\frac{x^2}{2}-2x$ $\bullet^2$ ✓ $\bullet^3$ ^

$=\frac{9}{2}$ $\bullet^4$ ✓

Candidate B - evidence of substitution using a calculator

$\bullet^1$ ^

$=-\frac{x^3}{3}+\frac{x^2}{2}-2x$ $\bullet^2$ ✓

$=\frac{10}{3}-\left(-\frac{7}{6}\right)$ $\bullet^3$ ✓

$=\frac{9}{2}$ $\bullet^4$ ✓

Candidate C - communication in Method 2

$\vdots$

$=-\frac{9}{2}$

Hence area is $\frac{9}{2}$ $\bullet^4$ ✓

Candidate D - communication in Method 2

$\vdots$

$=-\frac{9}{2}$

$=\frac{9}{2}\left(\text{units}^2\right)$ $\bullet^4$ ×

$=-\frac{9}{2}$ but area can't be negative

so $\frac{9}{2}\left(\text{units}^2\right)$ $\bullet^4$ ✓

$=-\frac{9}{2}$ but area can't be negative

$=\frac{9}{2}\left(\text{units}^2\right)$ $\bullet^4$ ×

$=-\frac{9}{2}$

so $\frac{9}{2}\left(\text{units}^2\right)$ as it's under the x-axis $\bullet^4$ ✓

$=-\frac{9}{2}$

so $\frac{9}{2}\left(\text{units}^2\right)$ $\bullet^4$ ×

Candidate E - limits not stated initially

$\bullet^1$ ^

$=-\frac{x^3}{3}+\frac{x^2}{2}-2x$ $\bullet^2$ ✓

$=\left(-\frac{(2)^3}{3}+\frac{(2)^2}{2}+2(2)\right)$

$-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2(-1)\right)$ $\bullet^3$ ✓

$=\frac{9}{2}$ $\bullet^4$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		$\bullet^1$ find the midpoint of AB $\bullet^2$ calculate m_{AB} $\bullet^3$ state perpendicular gradient $\bullet^4$ find equation of L	$\bullet^1$ $(4,-1)$ $\bullet^2$ -2 $\bullet^3$ $\frac{1}{2}$ $\bullet^4$ $y=\frac{1}{2}x-3$	**4**

Notes:

1. $\bullet^4$ is only available to candidates who find and use a perpendicular gradient and a midpoint.
2. The gradient of the perpendicular bisector must appear in fully simplified form at $\bullet^3$ or $\bullet^4$ stage for $\bullet^4$ to be awarded - see Candidate A.

Commonly Observed Responses:

Candidate A - unsimplified gradient

$m=-\frac{24}{12}$ $\bullet^2$ ✓

$m_{\perp}=\frac{12}{24}$ $\bullet^3$ ✓

$24y=12x-72$ $\bullet^4$ ^

Candidate B - using midpoint and m_{AC}

$(4,-1)$ $\bullet^1$ ✓

$m_{AC}=-1$ $\bullet^2$ ×

$m_{\perp}=1$ $\bullet^3$ ✓$_1$

$y=x-5$ $\bullet^4$ ✓$_1$

Candidate C

$m=-2=\frac{1}{2}$ $\bullet^2$ ✓ $\bullet^3$ ×

$y=\frac{1}{2}x-3$ $\bullet^4$ ✓$_1$ **- BOD**

Candidate D

$m=-2$ $\bullet^2$ ✓

$=\frac{1}{2}$ $\bullet^3$ ✓ **- BOD**

$y=\frac{1}{2}x-3$ $\bullet^4$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)		**Method 1** •5 calculate angle between one line and the x-axis •6 calculate angle between second line and the x-axis **AND** calculate angle between L and AC **Method 2** •5 calculate one angle within small triangle •6 calculate second angle within small triangle AND calculate angle between L and AC	**Method 1** •5 26.565…° **OR** 135° •6 26.565…° **OR** 135° leading to (135 − 26.565… =) 108.434…° **Method 2** •5 26.565…° **OR** 45° •6 26.565…° **OR** 45° leading to (180 − 45 − 26.565… =) 108.434…°	**2**

Notes:

3. Answers may be in radians: 0.4636…, and $\frac{3\pi}{4}$ (2.356…) leading to 1.8925… radians.
4. Do not penalise the omission of units at •6.
5. Accept any answers which round to 108° or 1.9 radians.
6. For 108° without working award 0/2.

Commonly Observed Responses:

Candidate E - Incorrect working

$\tan^{-1}(-1) = 45°$ •5 ×

Candidate F - Incorrect working

$\tan(\theta) = -1$

$\theta = 45°$ •5 ×

Candidate G - Poor notation

$m = -1$

$m = \tan\theta$

$\theta = \tan^{-1}(1)$

$\theta = 45$

$\theta = 180 - 45 = 135$ •5 ✓

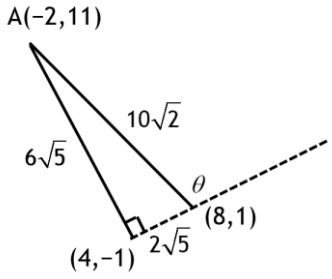
Candidate H - insufficient communication

$\theta = \tan^{-1}(1)$

$\theta = 45$

$180 - 45 = 135$ •5 ^

Candidate I - Using point of intersection, Pythagoras and right-angled trigonometry



•5 ✓

$\theta = 180 - \sin^{-1}\left(\frac{6\sqrt{5}}{10\sqrt{2}}\right) = 108.4°$ •6 ✓

Only two of the distances are required in the diagram and other trig ratios may be used.

Candidate J - Representing gradients as vectors

$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\cos\theta = \frac{-1}{\sqrt{5}\sqrt{2}}$ •5 ✓

$\theta = 108.4°$ •6 ✓

Any multiples of the vectors may be used

Question	Generic scheme	Illustrative scheme	Max mark
8. (b) (continued)			
Commonly Observed Responses (continued):			
Candidate K - Using a formula for angle between lines Acute angle between 2 lines = $\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$ $= \tan^{-1}\left(\frac{\frac{1}{2} - (-1)}{1 + (-1)\left(\frac{1}{2}\right)}\right)$ •5 ✓ $= \tan^{-1}(3)$ $= 71.6°$ Obtuse angle = $180 - 71.6 = 108.4°$ •6 ✓			

Question	Generic scheme	Illustrative scheme	Max mark
9.	**Method 1** $\bullet^1$ differentiate one non-constant term $\bullet^2$ complete differentiation and interpret condition $\bullet^3$ determine zeros of quadratic expression $\bullet^4$ state range with justification **Method 2** $\bullet^1$ differentiate one non-constant term $\bullet^2$ complete differentiation and determine zeros of quadratic expression $\bullet^3$ construct nature table(s) $\bullet^4$ interpret sign of derivative and state range	**Method 1** $\bullet^1$ $12x^2 \ldots$ **OR** $\ldots -24x$ $\bullet^2$ $12x^2 - 24x < 0$ $\bullet^3$ 0 **AND** 2 $\bullet^4$ $0 < x < 2$ with eg labelled sketch:  **Method 2** $\bullet^1$ $12x^2 \ldots$ **OR** $\ldots -24x$ $\bullet^2$ $12x^2 - 24x$ AND 0 and 2 . $\bullet^3$ $\bullet^4$ decreasing when $f'(x) < 0$ so $0 < x < 2$	**4**

	...	0	...	2	...
$f'(x)$	+	0	−	0	+

Notes:

1. At $\bullet^3$, do not penalise candidates who fail to extract the common factor or who have divided the quadratic inequality by 12.
2. $\bullet^3$ and $\bullet^4$ are not available to candidates who arrive at a linear expression at $\bullet^2$.
3. Accept the appearance of 0 and 2 within inequalities for $\bullet^3$.
4. In Method 2, $\bullet^3$ and $\bullet^4$ are not available where only one stationary point has been considered.
5. At $\bullet^4$, accept "$x > 0$, $x < 2$" together with the required justification.

Commonly Observed Responses:

Candidate A

$12x^2 - 24x < 0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$12x^2 - 24x = 0$

$x = 0,\ 2$ $\bullet^3$ ✓

$0 < x < 2$ with sketch $\bullet^4$ ✓

Candidate B - no initial inequation

$12x^2 - 24x = 0$ $\bullet^1$ ✓ $\bullet^2$ ×

$x = 0,\ 2$ $\bullet^3$ ✓

$0 < x < 2$ with sketch $\bullet^4$ ×

Question	Generic scheme	Illustrative scheme	Max mark
9. (continued)			
Commonly Observed Responses (continued):			
Candidate C Decreasing when $f'(x)<0$ $f'(x)=12x^2-24x$ $\bullet^1$ ✓ $\bullet^2$ ✓ ⋮		**Candidate D - condition applied after simplification** $f'(x)=12x^2-24x$ $\bullet^1$ ✓ $x^2-2x<0$ $\bullet^2$ ^ $x=0,\ 2$ $\bullet^3$ ✓ $0<x<2$ with sketch $\bullet^4$ ✓	

Question			Generic scheme	Illustrative scheme	Max mark
10.			$\bullet^1$ use double angle formula to express equation in terms of $\cos x°$	$\bullet^1$ $3\left(2\cos^2 x° - 1\right) + 10\cos x° \left(= 1\right)$	**5**
			$\bullet^2$ arrange in standard quadratic form	$\bullet^2$ $6\cos^2 x + 10\cos x - 4 = 0$	
			$\bullet^3$ factorise or use quadratic formula	$\bullet^3$ $2\left(3\cos x° - 1\right)\left(\cos x° + 2\right)\left(= 0\right)$ **OR** $\cos x = \dfrac{-10 \pm \sqrt{196}}{12}$	
			$\bullet^4$ solve for $\cos x°$	$\bullet^4$ $\cos x° = \dfrac{1}{3}$ $\quad$ $\bullet^5$ $\cos x° = -2$	
			$\bullet^5$ solve for x	$\bullet^5$ 70.52…, 289.47… 'no solutions'	

Notes:

1. Do not penalise the omission of the degree sign.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $1 - 2\sin^2 x°$ being substituted for $\cos 2x°$, $\bullet^1$ cannot be awarded until the equation reduces to a quadratic in $\cos x°$.
3. Substituting $2\cos^2 A - 1$ or $2\cos^2 \alpha - 1$ for $\cos 2x°$ at the $\bullet^1$ stage should be treated as bad form only where the equation is written in terms of x at $\bullet^2$ stage. Otherwise, $\bullet^1$ is not available.
4. "$= 0$" must appear by $\bullet^3$ stage for $\bullet^2$ to be awarded. However, for candidates using the quadratic formula to solve the equation, "$= 0$" must appear at the $\bullet^2$ stage for $\bullet^2$ to be awarded.
5. The equation $6\cos^2 x° - 4 + 10\cos x° = 0$ does not gain $\bullet^2$ unless $\bullet^3$ has been awarded.
6. $\bullet^3$ can be awarded for identifying the factors of the quadratic obtained at $\bullet^2$.
   For example, '$6\cos x° - 2 = 0$ and $\cos x° + 2 = 0$'.
7. Do not penalise candidates who fail to extract a common factor or who have divided the equation by 2 at $\bullet^3$.
8. Candidates may express their equation, obtained at $\bullet^2$, in the form $6c^2 + 10c - 4 = 0$ or $6x^2 + 10x - 4 = 0$. In these cases, award $\bullet^3$ for $2\left(3c - 1\right)\left(c + 2\right) = 0$, $2\left(3x - 1\right)\left(x + 2\right) = 0$ or equivalent. However, $\bullet^4$ is only available if $\cos x°$ appears explicitly at that stage. See Candidate A.
9. $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$ or for erroneously taking out a 'common factor' of $\cos x°$. See Candidate B.
10. $\bullet^4$ and $\bullet^5$ are only available as a consequence of trying to solve a quadratic equation. See Candidate C. However, $\bullet^5$ is not available if the quadratic has repeated roots or leads to no solutions for x.
11. Accept $x = \cos^{-1}\left(\dfrac{1}{3}\right)$ **AND** $x = \cos^{-1}\left(-2\right)$ for $\bullet^4$.
12. $\bullet^5$ is only available if one of the equations has no solutions. See Candidate E.
13. Ignore solutions outwith the range.
14. Accept solutions which round to 71 and 289, and ~~$\cos x° = -2$~~ at $\bullet^5$. Do not accept "error" at $\bullet^5$.

Question	Generic scheme	Illustrative scheme	Max mark

10. (continued)

Commonly Observed Responses:

Candidate A

$3\left(2\cos^2 x°-1\right)+10\cos x°=1$ $\bullet^1$ ✓

$6\cos^2 x+10\cos x-4=0$ $\bullet^2$ ✓

$6c^2+10c-4=0$

$2\left(3c-1\right)\left(c+2\right)=0$ $\bullet^3$ ✓

$c=\frac{1}{3},\ c=-2$ $\bullet^4$ ×

$x=70.5,\ 289.5$ $\bullet^5$ ×

Where candidates show ~~$c=-2$~~ award $\bullet^5$

Candidate B - not in standard quadratic equation form

$3\left(2\cos^2 x°-1\right)+10\cos x°=1$ $\bullet^1$ ✓

$6\cos^2 x°+10\cos x°=4$ $\bullet^2$ ×

$2\cos x°\left(3\cos x°+5\right)=4$ $\bullet^3$ × $\bullet^4$ × $\bullet^5$ ×

$\cos x°=2,\ \cos x°=-\frac{1}{3}$

No solutions, $x=109.5,\ 250.5$

Candidate C - not solving a quadratic equation

$3\left(2\cos^2 x°-1\right)+10\cos x°=1$ $\bullet^1$ ✓

$6\cos^2 x°+10\cos x°-4=0$ $\bullet^2$ ✓

$16\cos x°-4=0$ $\bullet^3$ × $\bullet^4$ × $\bullet^5$ ×

$\cos x°=\frac{1}{4}$

$x=75.5,\ 284.5$

Candidate D - reading $\cos 2x°$ as $\cos^2 x°$

$3\cos^2 x°+10\cos x°=1$ $\bullet^1$ × $\bullet^2$ ×

$3\cos^2 x°+10\cos x°-1=0$

$\cos x°=\frac{-10\pm\sqrt{112}}{6}$ $\bullet^3$ $✓_1$

$\cos x°=0.097,\ -3.431$ $\bullet^4$ $✓_1$

$x=84.4,\ 275.6$, no solutions $\bullet^5$ $✓_1$

Candidate E - incorrect factorising

⋮ $\bullet^1$ ✓ $\bullet^2$ ✓

$2\left(\cos x°-1\right)\left(\cos x°+2\right)=0$ $\bullet^3$ ×

$\cos x°=1,\ \cos x°=-2$ $\bullet^4$ $✓_1$

$x=0$, no solutions $\bullet^5$ $✓_1$

Candidate F - poor notation

$3\left(2\cos^2 x°-1\right)+10\cos x°=1$ $\bullet^1$ ✓

$6\cos^2 +10\cos x-4=0$ $\bullet^2$ ^

$2\left(3\cos x°-1\right)\left(\cos x°+2\right)=0$ $\bullet^3$ $✓_1$

Question			Generic scheme	Illustrative scheme	Max mark
11.			**Method 1** •[1] state linear equation •[2] introduce logs •[3] use laws of logs •[4] use laws of logs •[5] state a and b	**Method 1** •[1] $\log_4 y = 2x + 3$ •[2] $\log_4 y = 2x\log_4 4 + 3\log_4 4$ •[3] $\log_4 y = \log_4 4^{2x} + \log_4 4^3$ •[4] $\log_4 y = \log_4\left(4^{2x}\cdot 4^3\right)$ •[5] $a = 64$ **AND** $b = 16$ **OR** $y = 64\times 16^x$	**5**
			Method 2 •[1] state linear equation •[2] convert to exponential form •[3] use laws of indices •[4] state a •[5] state b	**Method 2** •[1] $\log_4 y = 2x + 3$ •[2] $y = 4^{2x+3}$ •[3] $y = 4^{2x}\cdot 4^3$ •[4] $a = 64$ •[5] $b = 16$ **OR** $y = 64\times 16^x$	
			Method 3 •[1] introduce logs to $y = ab^x$ •[2] use laws of logs •[3] interpret intercept •[4] interpret gradient •[5] state a and b	**Method 3** The equations at •[1], •[2], •[3] and •[4] must be stated explicitly •[1] $\log_4 y = \log_4\left(ab^x\right)$ •[2] $\log_4 y = \log_4 a + x\log_4 b$ •[3] $\log_4 a = 3$ •[4] $\log_4 b = 2$ •[5] $a = 64$ **AND** $b = 16$ **OR** $y = 64\times 16^x$	

Question	Generic scheme	Illustrative scheme	Max mark

11. (continued)

Notes:

1. In any method, marks may only be awarded within a valid strategy using $y = ab^x$.
2. Accept $y = 64.16^x$ for $\bullet^5$ in Methods 1 and 3, and for $\bullet^4$ and $\bullet^5$ in Method 2.
3. Markers must identify the method which best matches the candidate's approach; they must not mix and match between methods.
4. Penalise the omission of base 4 at most once in any method.
5. Where a candidate uses an incorrect base then only $\bullet^2$ and $\bullet^3$ are available (in any method).
6. Do not penalise the omission of brackets at $\bullet^4$ in Method 1 or $\bullet^1$ in Method 3.
7. For a correct answer with no working, award 1/5.

Commonly Observed Responses:

Candidate A - Use of coordinate pairs

$x = 3$ and $\log_4 y = 9$ $\bullet^1$ ✓

$\Rightarrow x = 3$ and $y = 4^9$ $\bullet^2$ ✓

$x = 0$ and $\log_4 y = 3$

$\Rightarrow x = 0$ and $y = 4^3$ $\bullet^3$ ✓

$4^3 = ab^0$ so $a = 64$ $\bullet^4$ ✓

$4^9 = 64b^3$ so $b = 16$ $\bullet^5$ ✓

Candidate B - ambiguous notation

$\log_4 y = \log_4 ab^x$ $\bullet^1$ ✓

$\log_4 y = \log_4 a + \log_4 bx$

$\log_4 a = 3$ $\bullet^3$ ✓

$\log_4 b = 2$ $\bullet^2$ ✓ $\bullet^4$ ✓

$y = 64 \times 16^x$ $\bullet^5$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		$\bullet^1$ differentiate two non-constant terms $\bullet^2$ complete differentiation and equate to zero $\bullet^3$ solve for x	$\bullet^1$ eg $3x^2+4x$ $\bullet^2$ $3x^2+4x-4=0$ $\bullet^3$ $\frac{2}{3}$, -2	**3**

Notes:

1. For a numerical approach, award 0/3.
2. $\bullet^2$ is only available if '$=0$' appears at $\bullet^2$ stage or in the working leading to $\bullet^3$. However, see Candidate A.
3. $\bullet^3$ is only available for solving a quadratic equation.

Commonly Observed Responses:

Candidate A - derivative not explicitly equated to 0

Stationary points occur when $f'(x)=0$

$f'(x)=3x^2+4x-4$ $\bullet^1$ ✓ $\bullet^2$ ✓

$f'(x)=(3x-2)(x+2)$

$x=\frac{2}{3}, -2$ $\bullet^3$ ✓

Candidate B - derivative never equated to 0

$3x^2+4x-4$ $\bullet^1$ ✓ $\bullet^2$ ^

$(3x-2)(x+2)$

$x=\frac{2}{3}, -2$ $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		•4 evaluate $f(x)$ at stationary point •5 consider value of $f(x)$ at end points •6 state maximum **AND** minimum values	•4 $-\frac{13}{27}$ or –0.481… or $\left(\frac{2}{3},-\frac{13}{27}\right)$ •5 $f(1)=0$, $f(-1)=6$ •6 max (value) $=6$ **AND** min (value) $=-\frac{13}{27}$	**3**

Notes:

4. 'Greatest (–1,6); least $\left(\frac{2}{3},-\frac{13}{27}\right)$' does not gain •6.
5. •4 is not available for only evaluating y at a value of x, obtained at the •3 stage, which lies outwith the interval $-1\le x\le 1$.
6. •5 is not available for evaluating y at an end point which is also identified as a stationary point in part (a).
7. •6 is only available where candidates have attempted to evaluate y at stationary point(s) in the interval $-1\le x\le 1$ identified in part (a).
8. •6 is only available where candidates have attempted to evaluate y at $x=-1$ and $x=1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•1 use compound angle formula •2 compare coefficients •3 process for k •4 process for a and express in required form	•1 $k\sin x°\cos a° - k\cos x°\sin a°$ stated explicitly •2 $k\cos a° = 2$ **AND** $k\sin a° = 7$ stated explicitly •3 $k = \sqrt{53}$ •4 $\sqrt{53}\sin(x - 74.05)°$	4

Notes:

1. Do not penalise the omission of degree symbols.
2. Accept $k(\sin x°\cos a° - \cos x°\sin a°)$ for •1.
3. Treat $k\sin x°\cos a° - \cos x°\sin a°$ as bad form only if the equations at the •2 stage both contain k.
4. $\sqrt{53}\sin x°\cos a° - \sqrt{53}\cos x°\sin a°$ **or** $\sqrt{53}(\sin x°\cos a° - \cos x°\sin a°)$ are acceptable for •1 and •3.
5. •2 is not available for $k\sin x° = 7$ and $k\cos x° = 2$. However, •4 may still be gained. See Candidate F.
6. Accept $-k\sin a° = -7$ and $k\cos a° = 2$ for •2.
7. •4 is not available for a value of a given in radians.
8. Accept values of a which round to 74.
9. Candidates may **state and use** any form of the wave function for •1, •2 and •3. However, •4 is only available if the wave is interpreted in the form $k\sin(x - a)°$.
10. Evidence for •4 may not appear until part (b) and must appear by the •5 stage.

Commonly Observed Responses:

Candidate A

$\sqrt{53}\cos a° = 2$ •1 ^

$\sqrt{53}\sin a° = 7$ •2 ✓ •3 ✓

$\tan a° = \frac{7}{2}$

$a = 74.05...$

$\sqrt{53}\sin(x - 74.05...)°$ •4 ✓

Candidate B

$k\sin x°\cos a° - k\cos x°\sin a°$ •1 ✓

$\cos a° = 2$

$\sin a° = 7$ •2 ×

$\tan a° = \frac{7}{2}$

$a = 74.05...$ (not consistent with equations at •2)

$\sqrt{53}\sin(x - 74.05...)°$ •3 ✓ •4 ×

Candidate C - missing *k*

$\sin x°\cos a° - \cos x°\sin a°$ •1 ×

$\cos a° = 2$

$\sin a° = 7$ •2 ×

$k = \sqrt{53}$ •3 ✓

$\tan a° = \frac{7}{2}$

$a = 74.05...$ (not consistent with equations at •2)

$\sqrt{53}\sin(x - 74.05...)°$ •4 ×

Candidate D - errors at •2

$k\sin x°\cos a° - k\cos x°\sin a°$ •1 ✓

$k\cos a° = 7$

$k\sin a° = 2$ •2 ×

$\tan a° = \frac{2}{7}$

$a = 15.94...$

$\sqrt{53}\sin(x - 15.94...)°$ •3 ✓ •4 $✓_1$

Question	Generic scheme	Illustrative scheme	Max mark

13. (a) (continued)

Commonly Observed Responses (continued):

Candidate E - errors at $\bullet^2$

$k\sin x^\circ\cos a^\circ - k\cos x^\circ\sin a^\circ$ $\bullet^1$ ✓

$k\cos a^\circ = 2$

$k\sin a^\circ = -7$ $\bullet^2$ ×

$\tan a^\circ = -\frac{7}{2}$

$a = 285.94...$

$\sqrt{53}\sin(x - 285.94\ldots)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\checkmark_1$

Candidate F - use of x at $\bullet^2$

$k\sin x^\circ\cos a^\circ - k\cos x^\circ\sin a^\circ$ $\bullet^1$ ✓

$k\cos x^\circ = 2$

$k\sin x^\circ = 7$ $\bullet^2$ ×

$\tan a^\circ = \frac{7}{2}$

$a = 74.05...$

$\sqrt{53}\sin(x - 74.05...)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\checkmark_1$

Candidate G

$k\sin A\cos B - k\cos B\sin A$ $\bullet^1$ ×

$k\cos A = 2$

$k\sin A = 7$ $\bullet^2$ ×

$\tan A = \frac{7}{2}$

$A = 74.05...$

$\sqrt{53}\sin(x - 74.05...)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\checkmark_1$

Candidate H - missing k

$\sin x^\circ\cos a^\circ - \cos x^\circ\sin a^\circ$ $\bullet^1$ ×

$k\cos a^\circ = 2$

$k\sin a^\circ = 7$ $\bullet^2$ ×

$\tan a^\circ = \frac{7}{2}$

$a = 74.05...$

$\sqrt{53}\sin(x - 74.05...)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\checkmark_1$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•5 form equation, link to (a) and rearrange •6 solve for $x-74.05\ldots$ •7 find x-coordinate of P	•5 $\sin(x-74.05)^\circ=\frac{3}{\sqrt{53}}$ •6 $x-74.05=$ 24.33 **OR** 155.66 **OR** −204.33 •7 −130.28	**3**

Notes:

11. Accept any value of x which rounds to −130.
12. •7 is only available for a single value of x. See Candidate L.
13. When awarding follow-through marks, •7 is only available for a single value of x between −180 and 0.

Commonly Observed Responses:

Candidate I - Follow through from COR D

$\sin(x-15.94\ldots)^\circ=\frac{3}{\sqrt{53}}$ •5 ✓1

$x-15.94=24.33$ (or 155.66) •6 ✓1

$x=-188.39\ldots$ •7 ✗

Candidate J - Follow through from COR E

$\sin(x-285.94\ldots)^\circ=\frac{3}{\sqrt{53}}$ •5 ✓1

$x-285.94=24.33$ (or 155.66) •6 ✓1

$x=-49.71\ldots$ •7 ✓1

Candidate K - Missing rearrangement

$\sqrt{53}\sin(x-74.05)^\circ=3$

$x-74.05=24.33$ •5 ✓ •6 ✓

Candidate L - multiple solutions

$\sin(x-74)^\circ=\frac{3}{\sqrt{53}}$ •5 ✓

$x-74.05=24.33, 155.66, -204.33$ •6 ✓

$x=98.39, 229.72, -130.28$ •7 ✗

However, where the final solution is clearly identified by the candidate, award •7

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•[1] calculate temperature	•[1] 825 (°C)	**1**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		$\bullet^2$ substitute for T $\bullet^3$ process equation $\bullet^4$ express in logarithmic form $\bullet^5$ solve for t	$\bullet^2$ $150 = 800 \times e^{-0.124t} + 25$ $\bullet^3$ $\frac{150-25}{800} = e^{-0.124t}$ $\bullet^4$ $\log_e \frac{150-25}{800} = -0.124t$ $\bullet^5$ 14.970 (seconds)	4

Notes:

1. Where values other than 150 are used in the substitution $\bullet^3$, $\bullet^4$ and $\bullet^5$ are still available.
2. $\bullet^3$ may be implied by $\bullet^4$.
3. Evidence for $\bullet^4$ must be stated explicitly. See Candidate B.
4. At $\bullet^4$ all exponentials must be processed.
5. Any base may be used at $\bullet^4$ stage, See Candidate A.
6. Accept $\ln 0.15625 = -0.124t \ln e$ and $-1.856\ldots = -0.124t$ for $\bullet^4$.
7. $\bullet^5$ is unavailable where candidates round the value of $\ln 0.15625$ to fewer than 2 decimal places.
8. Accept answers where $14.9 \le t \le 15$ at $\bullet^5$.
9. The calculation at $\bullet^5$ must follow from the valid use of exponentials and logarithms at $\bullet^3$ and $\bullet^4$.
10. Where candidates show no working, or take an iterative approach to arrive at $t = 14$ or $t = 15$, award 1/4. However, if, in any iterations T is evaluated for $t = 14.9$ and $t = 15$, leading to a final answer of $t = 15$ (seconds), then award 4/4. See Candidate C.

Commonly Observed Responses:

Candidate A - using other bases

$150 = 800 \times e^{-0.124t} + 25$ $\bullet^2$ ✓

$\frac{5}{32} = e^{-0.124t}$ $\bullet^3$ ✓

$\log_{10} \frac{5}{32} = -0.124t \log_{10} e$ $\bullet^4$ ✓

$t = 14.97$ $\bullet^5$ ✓

Candidate B - missing working

$150 = 800 \times e^{-0.124t} + 25$ $\bullet^2$ ✓

$0.15625 = e^{-0.124t}$ $\bullet^3$ ✓

$\bullet^4$ ^

$t = 14.97$ $\bullet^5$ ✓

Candidate C - iterative approach

$t = 13 \Rightarrow T = 184.59\ldots$

$t = 14 \Rightarrow T = 165.979\ldots$

$t = 14.9 \Rightarrow T = 151.091\ldots$

$t = 15 \Rightarrow T = 149.538\ldots$

so $t = 15$

Award 4/4

Candidate D - taking logarithms of both sides

$150 = 800 \times e^{-0.124t} + 25$ $\bullet^2$ ✓

$125 = 800 \times e^{-0.124t}$

$\log_e 125 = \log_e \left(800 \times e^{-0.124t}\right)$

$\log_e 125 = \log_e 800 + \log_e e^{-0.124t}$ $\bullet^3$ ✓

$\log_e 125 - \log_e 800 = -0.124t$ $\bullet^4$ ✓

$t = 14.97$ $\bullet^5$ ✓

Candidate E - transcription error

$150 = 800e^{-0.124t}$ $\bullet^2$ ✗

$0.1875 = e^{-0.124t}$ $\bullet^3$ ✓$_1$

$-0.124t = -1.673$ $\bullet^4$ ✓$_1$

$t = 13.5$ $\bullet^5$ ✓$_1$

Candidate F - Incorrect processing

$150 = 800 \times e^{-0.124t} + 25$ $\bullet^2$ ✓

$\log_e 150 = \log_e 800 \times e^{-0.124t} + 25$

$\frac{\log_e 150 - 25}{\log_e 800} = e^{-0.124t}$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗

Question	Generic scheme	Illustrative scheme	Max mark
14. (b) (continued)			
Commonly Observed Responses (continued):			
Candidate G - Incorrect processing $150 = 800 \times e^{-0.124t} + 25$ $\frac{150}{800} = e^{-0.124t} + 25$ $\frac{150}{800} - 25 = e^{-0.124t}$ $\log_e\left(\frac{150}{800} - 25\right) = -0.124t$	$\bullet^2$ ✓ $\bullet^3$ × $\bullet^4$ × $\bullet^5$ ×		

Question	Generic scheme	Illustrative scheme	Max mark
15.	$\bullet^1$ expand	$\bullet^1$ $3\sin^2\theta+9\sin\theta\cos\theta$ $+\cos\theta\sin\theta+3\cos^2\theta$	**3**
	$\bullet^2$ use appropriate trigonometric identity	$\bullet^2$ $3\ldots$ or $\ldots 5\sin 2\theta$	
	$\bullet^3$ use second trigonometric identity and express in required form	$\bullet^3$ $3+5\sin 2\theta$	

Notes:

1. $\bullet^1$ and $\bullet^2$ are available to candidates work with, for example, A or x in place of θ. However, $\bullet^3$ is not available.
2. Marks are not available to candidates for quoting $\sin^2\theta+\cos^2\theta=1$ or $\sin 2\theta=2\sin\theta\cos\theta$.

Commonly Observed Responses:

Candidate A - incorrect notation

$3\sin\theta^2+10\sin\theta\cos\theta+3\cos\theta^2$	$\bullet^1$ ✗
$3+10\sin\theta\cos\theta$	$\bullet^2$ ✗
$3+5\sin 2\theta$	$\bullet^3$ ✓1

Candidate B - incorrect notation

$3\sin^2\theta+10\sin\theta\cos\theta+3\cos\theta^2$	$\bullet^1$ ✗
$3+10\sin\theta\cos\theta$	$\bullet^2$ ✗
$3+5\sin 2\theta$	$\bullet^3$ ✓1

Candidate C - incorrect expansion leading to a non-integer value of *b*

$3\sin^2\theta+6\sin\theta\cos\theta+\cos\theta\sin\theta+3\cos^2\theta$	$\bullet^1$ ✗
$3\sin^2\theta+7\sin\theta\cos\theta+3\cos^2\theta$	
$3+7\sin\theta\cos\theta$	$\bullet^2$ ✓1
$3+3.5\sin 2\theta$	$\bullet^3$ ✗

[END OF MARKING INSTRUCTIONS]