H

Essential Skills
Higher
Mathematics
Practice Exam 2026

Paper 1 (Non-calculator)

Time 1 hour and 15 minutes

Total marks – 55

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks, you must show your working in your answers.

State the units for your answer where appropriate.

Use **blue** or **black** ink.

Marks available are harsh but I wanted as much course coverage as possible.

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin\mathrm{A}\cos\mathrm{B} \pm \cos\mathrm{A}\sin\mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos\mathrm{A}\cos\mathrm{B} \mp \sin\mathrm{A}\sin\mathrm{B}$$

$$\sin 2\mathrm{A} = 2\sin\mathrm{A}\cos\mathrm{A}$$

$$\cos 2\mathrm{A} = \cos^2\mathrm{A} - \sin^2\mathrm{A}$$

$$= 2\cos^2\mathrm{A} - 1$$

$$= 1 - 2\sin^2\mathrm{A}$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
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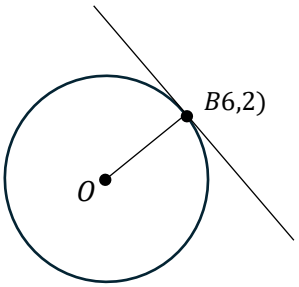
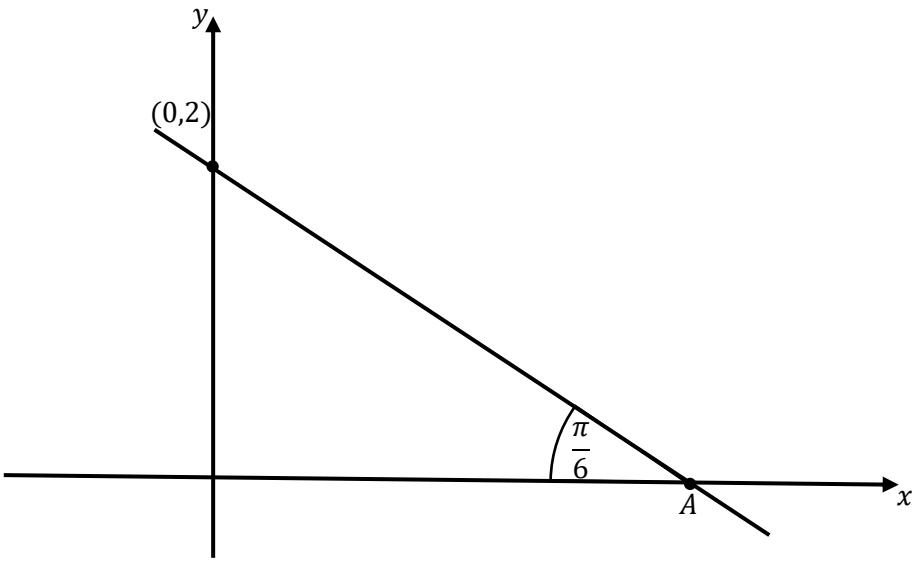
1. Point $B(6,2)$ lies on the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.



Find the equation of the tangent to the circle at B. **4**

2. A line passes through the point (0, 2) and makes an angle of $\frac{\pi}{6}$ with the x –axis as shown.



(a) Determine the equation of the line. **3**

(b) Establish the coordinates of the point A. **1**

3. A sequence is generated by the recurrence relation

$$U_{n+1} = mU_n + c$$

The first 3 values of the sequence are 25, 15 and 11.

(a) Find the values of m and c. **3**

(b) Calculate the limit of the sequence. **2**

4. The points $P(1, 2, -1), Q(k, -2, 1)$ and $R(7, -10, 5)$ are collinear.

(a) Calculate the value of k. **2**

(b) State the ratio that Q divides PR. **1**

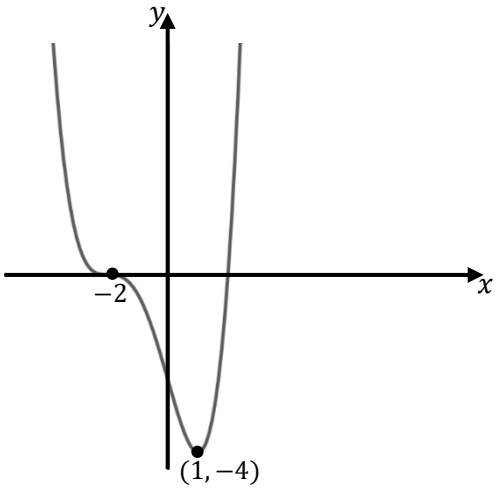
5. A function, f, is defined as $f(x) = x^3 + 1$, where $0 \leq x \leq 4$.

(a) Find the inverse function $f^{-1}(x)$. **3**

(b) State the domain of $f^{-1}(x)$. **1**

6. The diagram shows the graph of $y = f(x)$.

There is a minimum turning point at (1, -4) and a point of inflection at $x = -2$.



(a) Sketch the graph of $y = 2 - f(x + 1)$. **3**

(b) Sketch the graph of $y = f'(x)$. **2**

7. Express $x^3 + 2x + 3$ in fully factorised form. **3**

8. Given $g(x) = cos3x + sin3x$, find $g'(\frac{\pi}{2})$. **4**

9. Express $2log_a 3 + log_a 4 - 3log_a 2$ in the form $log_a\left(\frac{B}{C}\right)$, where B and C are whole numbers **3**

10. **(a)** Express $\sqrt{3}cosx° - sinx°$ in the form $kcos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

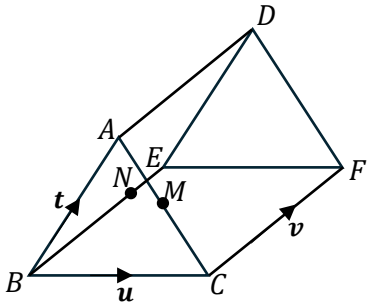
(b) Hence, or otherwise, find:

(i) the minimum value of $y = \sqrt{3}cosx° - sinx° + 1$ **1**

(ii) determine the value of a where it occurs $0 < a < 2\pi$. **2**

11. The diagram shows the triangular prism ABCDEF.

$\overrightarrow{BA} = \boldsymbol{t}$, $\overrightarrow{BC} = \boldsymbol{u}$ and $\overrightarrow{CF} = \boldsymbol{v}$.



(a) Express $\overrightarrow{AC}$ in terms of $\boldsymbol{t}$ and $\boldsymbol{u}$. **1**

M is the midpoint of AC; N divides BE in the ratio 3:1.

(b) Express $\overrightarrow{MN}$ in terms of $\boldsymbol{t}$, $\boldsymbol{u}$ and $\boldsymbol{v}$. **2**

12. The point $P(x, y)$ lies on the curve $y = 6x^2 - 2x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 6. **4**

(b) Hence find the equation of the tangent a P. **2**

13. Given that $tan2x = \frac{\sqrt{5}}{2}$, $0 < x < \frac{\pi}{4}$, find the exact value of:

(a) $cos2x$ **1**

(b) $cosx$ **2**

[END OF QUESTION PAPER]

Answers

1. $3x + 4y - 26 = 0$

2. (a) $\sqrt{3}y + x - 2\sqrt{3} = 0$ (b) $A(2\sqrt{3}, 0)$

3. (a) $m = \frac{2}{5}$ $c = 5$ (b) L = $\frac{25}{3}$

4. (a) $k = 3$ (b) $1:2$

5. (a) $f^{-1}(x) = \sqrt[3]{x-1}$ (b) $1 \leq x \leq 65$

6. (a) max TP at $(0,6)$; POI at $(-3,2)$ (b) max TP at $x = -2$, root at $x = 1$.

7. $(x+1)(x^2 - x + 3)$

8. 3

9. $log_a\left(\frac{9}{2}\right)$

10. (a) $2\cos\left(x + \frac{\pi}{6}\right)$ (b) (i) -1 (ii) $\frac{5\pi}{6}$

11. (a) $\overrightarrow{AC} = u - t$ (b) $\overrightarrow{MN} = \frac{3}{4}v - \frac{1}{2}t - \frac{1}{2}u$

12. (a) $x = 1$ (b) $y - 6x + 2 = 0$

13. (a) $cos2x = \frac{2}{3}$ (b) $cosx = \frac{\sqrt{30}}{6}$ or $cosx = \frac{\sqrt{5}}{\sqrt{6}}$

H

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Paper 2 (Calculator)

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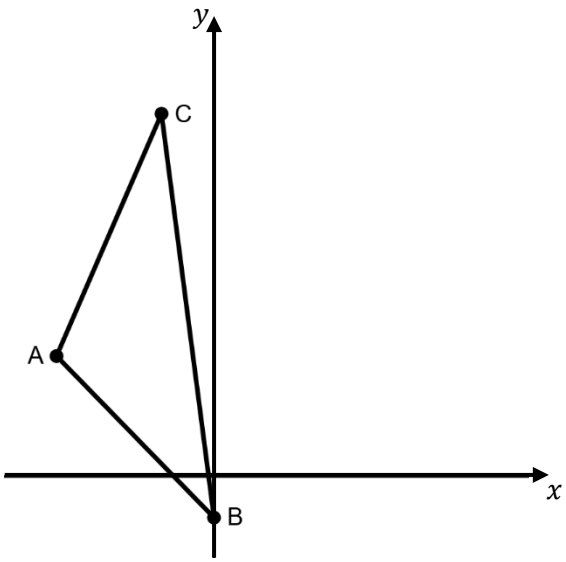
1. The vertices of a triangle are A(-6, 3), B(0, -1) and C(-2, 9) as shown.



(a) Find the equation of the perpendicular bisector of AB. **4**

(b) Determine the coordinates of the point where the perpendicular bisector of AB intersects side BC. **4**

2. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 6x - 4x^2$.

The curve passes through the point $(2,5)$.

Express y in terms of x. **4**

3. **(a)** Given $A(2,-1,4), B(6,3,1)$ and $C(-1,5,7)$,

express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2**

(b) Calculate the size of the angle BAC. **4**

4. Show that $x^2 + (m+3)x + (m-5) = 0$ has real roots for all values of m. **4**

5. Solve $log_3 x + log_3(x-8) = log_7 49$, where $x > 8$ **4**

6. A function, f, is defined by $f(x) = x^4 - 4x^3 + 5$.

Find the x-coordinates of the stationary points of f and determine their nature. **5**

7. Functions f and g are given by $f(x) = 11 - 8cosx$ and $g(x) = 2x$, $x \in \mathbb{R}$.

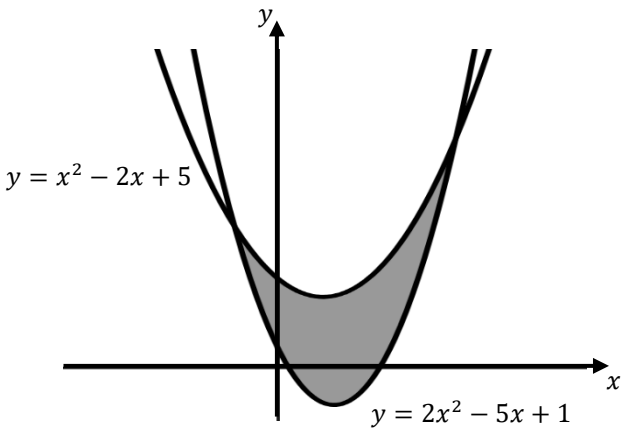
(a) Find expressions for:

(i) $f(g(x))$ **1**

(ii) $g(f(x))$ **1**

(b) Determine the values of x for which $f(g(x)) = g(f(x))$, where $0 \leq x \leq 360$. **5**

8. The graphs of $y = x^2 - 2x + 5$ and $y = 2x^2 - 5x + 1$ are shown below.



Calculate the shaded area. **6**

9. Show that $\frac{1-cos2x}{sin2x} = tanx$, where $0 < x < \frac{\pi}{2}$. **3**

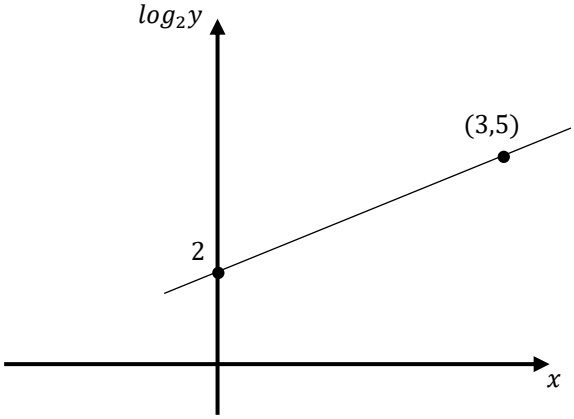
10. A cylindrical tin of Mintos has a surface area given by

$$A(r) = 2\pi r^2 + \frac{1200}{r}$$

Determine the value of r that will minimise the surface area. **5**

11. Two variables x and y are connected by the equation $y = ab^x$.

The graph of $log_2 y$ against x is a straight line passing through $(0,2)$ and $(3,5)$.



Find the values a and b. **5**

12. The circle C_1 has equation $(x-1)^2 + (y-2)^2 = 100$.

A circle, C_2, with equation $x^2 + y^2 - 2x + 4y + k = 0$ is drawn inside C_1.

The circles have no point of contact.

Find the range of values for k. **8**

[END OF QUESTION PAPER]

Answers

1. (a) $2y - 3x - 11 = 0$ (b) $(-1, 4)$

2. $y = 3x^2 - \frac{4}{3}x^3 + \frac{11}{3}$

3. (a) $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} \overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ (b) 86·3°

4. Proved; $(m+1)^2 + 28 \geq 0 \; \forall \; m$

5. $x = 9$

6. Decreasing point of inflection at $x = 0$, minimum turning point at $x = 3$

7. (a) (i) $11 - 8cos2x$ (ii) $22 - 16cosx$ (b) $x = 41{\cdot}4°, 75{\cdot}5°, 284{\cdot}5°, 318{\cdot}6°$

8. $\frac{125}{6}$ units²

9. proof

10. $r = 4 \cdot 57 \; units$

11. $a = 4, b = 2$

12. $-31 < k < 5$