

SENIOR FIVE
HOLIDAY PACKAGE — TERM II
PRINCIPAL MATHEMATICS 2 (P425/2)

ITEM 1

Mr. Mukasa is a coffee farmer who is planning for both his next planting season and the future needs of his family.

He intends to plant three varieties of coffee seeds, S_1 , S_2 and S_3 , in the ratio 4:2:1 respectively. Agricultural research indicates that the probabilities of germination for S_1 , S_2 and S_3 are 50%, 60% and 80%, respectively. Mr. Mukasa plans to plant 150 seeds and hopes to obtain at least 90 healthy seedlings.

Mr. Mukasa is also planning for his family. He expects to have six children. From available health records, the probability of having a girl is twice that of having a boy. According to the family's tradition, every boy is allocated a piece of land as starting capital when he marries. Mr. Mukasa currently has enough land to provide for two boys.

TASK

- (a) As a student of Mathematics, use appropriate mathematical justification to advise Mr. Mukasa whether the proposed coffee-seed mixture is reliable enough for his planting plan.

ITEM 2

Mr. Kato, a traffic officer in Mbarara, is studying traffic flow at a busy stage and reviewing cases of drivers who were wrongly penalised and later received compensation.

During rush hours, the number of vehicles leaving the stage in one hour follows a rectangular distribution with a mean of 12 vehicles and variance 3. The traffic department considers additional control measures necessary if the upper limit of the distribution exceeds 15 vehicles per hour.

The traffic police are also investigating driver responses at highway checkpoints. Their investigations have led to several court cases in which some drivers were later found not guilty, resulting in compensation being paid to them.

The response variable X , measured in appropriate units, has the probability density function:

$$f(x) = \begin{cases} 2k(x + 1) & ; -1 \leq x \leq 0 \\ k(2 - x) & ; 0 \leq x \leq 2 \\ 0 & ; \text{else where} \end{cases}$$

Most drivers have been taken to court but later found guilty of the offences which have caused the traffic police to compensate much money to these drivers. To reduce wrongful enforcement, the department requires all cases with response scores below the median to be reviewed before a penalty is issued. If the proportion of such cases is more than 50%, the current assessment procedure must be revised.

TASK

- Using appropriate mathematical evidence, determine whether additional traffic-control measures are justified during rush hour and advise Mr. Kato.
- Using appropriate mathematical evidence, determine whether the current driver-assessment procedure should be revised to reduce wrongful enforcement and compensation claims, and advise Mr. Kato.

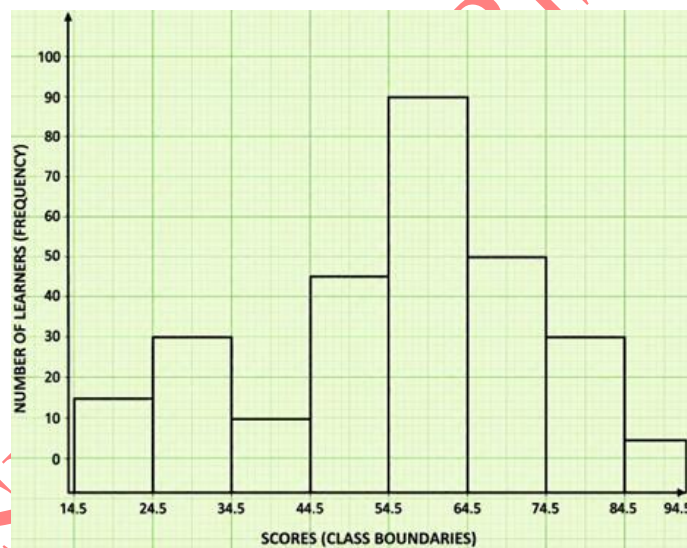
ITEM 3

Mr. Okello, the Head Teacher of a secondary school, employed a new Mathematics teacher to improve the performance of Senior Six learners.

Before the teacher was employed, the mean Mathematics score was 54%, and 38% of the learners scored below the school's pass mark of 55%.

After the new teacher had taught the class for a full term, the learners sat for an examination. Their results are represented in the graph below.

GRAPH 1: number of learners against mathematics scores



The school needs help to analyse the results of learners performance shown on the graph above and establish whether there is an improvement in the learners' performance basing on

- (i) The mean score of the results on graph 1 in comparison to the previous mean score.
- (ii) The percentage of learners below the school pass mark of 55%.

TASK

Provide the school with the help they need based on mathematics justifications.

ITEM 4

Eagle Shield Security Services Ltd is a rapidly growing security company that provides professional security services to high-profile clients, including banks, corporate organizations, and residential estates. As part of its recruitment process, the company conducts aptitude tests to identify applicants with the required skills and abilities to perform security duties effectively.

The company assesses applicants in two key areas: communication skills and analytical skills. The scores obtained by ten shortlisted applicants are shown below.

Applicant	1	2	3	4	5	6	7	8	9	10
Communication skills (x)	40	90	54	32	80	65	55	48	55	30
Analytical skills (y)	68	40	47	64	55	41	62	76	74	80

During a management meeting, some members argued that applicants with excellent communication skills are likely to have strong analytical abilities, while others disagreed. The management therefore wants to establish whether there is any relationship between the two skills.

After reviewing the academic records of applicants, the Human Resource department obtained the following information: The probability that a randomly selected applicant is female is $\frac{1}{2}$. The probability that a female applicant possesses an O'level certificate is $\frac{1}{3}$. The probability that a male applicant possesses an O'level certificate is $\frac{1}{4}$.

The company intends to increase the number of male security guards recruited, but only if the probability that a randomly selected applicant who does not possess an O'level certificate is male exceeds 0.5.

Task

- (a) Help the management to determine whether communication skills and analytical skills are associated among the shortlisted applicants.
- (b) Determine whether the Human Resource department should proceed with its plan to recruit more male security guards.

ITEM 5

In a secondary school, the issuance of school uniforms for Senior One students depends on their heights. As they proceed to the successive classes, their uniform sizes keep changing and new uniforms need to be issued again. It is the school policy that only when the initial mean height is at least 125 cm will new uniforms be issued.

The heights are as follows:

Height(cm)	100-110	110-120	120-130	130-140	140-150	150-160	160-170
Cumulative frequency	27	85	215	320	370	395	400

Sometimes the head teacher prefers storing the heights of new Senior One students in a graphical format to easily establish the most common height initially on admission to guide on the amount of material to purchase as students progress in the next classes.

On the reporting day, the school recorded the number of students who successfully received their new uniforms. The number of students who obtained uniforms in a given hour is represented by the discrete random variable, with the following probability distribution:

Number of students who obtained uniforms (x)	0	1	2	3	4	5
$P(X = x)$	0.05	0.15	0.25	0.30	0.20	0.05

If the expected number of students receiving uniforms in one hour exceeds three, the head teacher will recruit an additional person to assist in issuing uniforms.

Task

As a mathematics student, if consulted, advise the school to establish:

- Whether new uniforms will or will not be issued.
- How the head teacher will store the data and also highlight for him the most likely height to enable the change of uniform.
- Whether the Headteacher will recruit a person to assist in issuing uniforms.

ITEM 6

A construction company in Kasese District purchased a heavy-duty truck to transport building materials to remote areas. Before using the truck, engineers carried out tests to determine whether it could safely operate under different road conditions.

The truck has a mass of 2000 kg. During the first test, it moves along a rough horizontal road where the coefficient of friction between the tyres and the road surface is 0.3. The engine provides a constant driving force of 8000 N.

After successfully completing the first test, the engineers move the truck to a mountainous area where the road is inclined at to the horizontal. Due to the rough gravel surface on the slope, the coefficient of friction between the tyres and the inclined road is 0.4. The truck operates at a higher engine output and experiences a driving force of 16000 N as it moves up the slope.

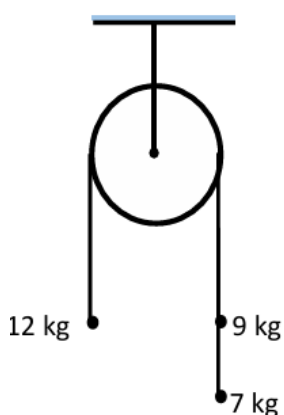
The construction company will recommend the use of this type of truck in other remote areas if the acceleration of the truck in mountainous areas is greater than its acceleration on the horizontal road.

Task

Determine whether the construction company would recommend the use of trucks.

ITEM 7

Mr. Brian, the Physics laboratory technician, is preparing a practical experiment involving three masses of 12 kg, 9 kg and 7 kg connected by light inextensible strings as shown in the diagram.



The string connecting the 12 kg and 9 kg masses passes over a smooth fixed pulley, while the second string connects the 9 kg and 7 kg masses. When the system is released, the 12 kg mass moves upwards.

The laboratory has two types of strings available. One has a maximum safe tension of 120 N, while the other has a maximum safe tension of 150 N.

Mr. Brian wants to ensure that the practical experiment is conducted safely.

TASK

- As a student of Mathematics, use appropriate mathematical principles to advise Mr. Brian which string should be used for the experiment.
- Give a mathematical justification for your recommendation.

ITEM 8

Mr. Samuel is a structural engineer working on the design of a regular hexagonal building ABCDEF. Strong winds are expected in the area, and several forces act on the building.

Forces of magnitude 3 N, 4 N, 2 N and 6 N act along AB, AC, EA and AF, respectively, with the direction of each force indicated by the order of the letters.

AB and AE are taken as the horizontal and vertical axes respectively.

Mr. Samuel plans to install a resistant concrete column to counteract the effect of the wind forces and prevent movement of the building.

TASK

- (a) As a student of Mathematics, investigate the wind forces and advise Mr. Samuel on the force and direction required to keep the building in equilibrium.
- (b) Give a mathematical justification for your recommendation.

ITEM 9

Mr. David, the school maintenance officer, is checking a 450 g wooden bar resting on a rough horizontal surface. During maintenance, the bar is subjected to forces of 2 N, 3 N, 4 N, 5 N and 6 N in directions 030° , 090° , 120° , 210° and 330° , respectively. The coefficient of friction between the bar and the surface is 0.40.

A second wooden structure is subjected to forces of 6 N, 5 N, 7 N, 8 N, 5 N and 9 N in directions $N30^\circ E$, $N30^\circ W$, $S50^\circ E$, $N60^\circ W$, $N60^\circ E$ and $S40^\circ W$, respectively. Mr. David has a 5 N support machine available to control the structure.

TASK

As a student of Mathematics, use appropriate mathematical principles to:

- (a) Determine whether the first wooden bar is likely to remain at rest or move. If it moves, determine its acceleration.
- (b) Advise Mr. David with mathematical justification, whether the 5 N machine is sufficient to keep it stable.

ITEM 10

Mr. Musa is a road-safety officer investigating stopping distances on a busy highway.

A car travelling at a steady speed of 25 m s^{-1} approaches a fallen tree 65 m ahead. The driver immediately applies the brakes, producing a constant retardation of 5 m s^{-2} .

The road-safety authority requires vehicles to stop with a safety margin of at least 5 m from an obstacle.

Mr. Musa is also concerned about situations where drivers take time to react. He wants to understand the consequences if the same driver delays braking for one second.

TASK

- (a) As a student of Mathematics, use appropriate mathematical principles to advise Mr. Musa whether the car can safely stop before the fallen tree when the driver brakes immediately.
- (b) Based on mathematical evidence, advise Mr. Musa on the safety implications of a one-second delay in applying the brakes.

ITEM 11

A civil engineering firm is analyzing the forces acting on a regular, two-dimensional steel truss component used in a pedestrian bridge design. The truss structure is shaped like a regular hexagon, PQRSTU, with each side measuring 2m.

The structure is subjected to six external forces due to wind, gravity, and tensioning cables. These forces and their directions, indicated by the order of the letters of the truss, are as follows: 9N, 5N, 7N, 3N, 1N and 4N act in PQ, QR, RS, ST, TU and UP.

The engineering team needs a complete vector analysis to ensure the truss can withstand the resultant load. If the resultant force exceeds 6N and its direction with the horizontal is less than 60° the system will not be stable.

Task

Help the engineering team to determine whether the system will be stable.

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