

Uganda Advanced Certificate of Education

S.5 Pure Mathematics Holiday

Item 1

The academic committee of a certain school is planning an inter-at and class quiz. The H/M who is the chairperson of the meeting committee wishes to justify the budget.

The committee has shs. 470,000/= for 3 quiz books, 2 traditional attire sets and 4 trophies.

1 set of attires and 1 trophy together cost shs. 60,000/= more than 2 quiz books.

A trophy costs shs. 40,000/= more than a quiz book.

The committee has been given extra shs. 150,000/= and wishes to get 2 more sets of attires and 1 quiz book if the extra money given to them is enough.

In preparations for the quiz, the headteacher called for two meetings. In the first meeting, they spent 5^{x+2} on drinks and 7^{y+1} on snacks and they spent 3468 altogether.

In the last meeting, the amount spent on snacks was 7^y and this was exactly 76 less than the amount spent on drinks which was 5^x . The committee wishes to solicit for more funds if the difference between the two unknowns is less than five.

On the day of the quiz, the committee wishes to hire rectangular tables. According to the supplier, the cost $C(x)$ of one table is given by $ax^3 + bx^2 + 5x + 20$ where x is the width of the table in metres. For a 1 m wide table, the cost is 29 and for a 2 m wide table, the cost is 48 in thousands. The committee has allocated shs. 130,000/= for a 4 m wide table and wishes to hire it if the money allocated for it is enough.

Task:

Using appropriate mathematical methods, advise the committee with justification on the decisions to take.

Item 2

Alex and Francis are friends that always have daily arguments starting from the premier league teams they support that is, Manchester United and Arsenal respectively. Recently they were moving around school and entered a S.4

classroom where they found an expression on the chalkboard reading $\sin 3A$, immediately the argument started when Alex claimed that the handwriting looked familiar and Francis said it was a lie. They later moved to the next classroom of S.3 and found another expression on the white board reading $3 \sin A - 4 \sin^3 A$, another argument came in when Francis said that the two statements look different but mean the same thing, "no way, it cannot be, lets bet on it buddy", were the statements of Alex.

As they were still arguing, Francis received a notification on his phone from BBC news showing that Iran was to launch a nuclear missile over Israel whose speed is given by the expression $s = 3 \cos \theta + \sin \theta$ where s is the speed in kilometres per hour (km/h) and θ the angle of projection.

Tasks

Using the knowledge of trigonometry help these two;

- Without using calculators or tables, settle the argument whether the statements mean the same thing or not.
- Determine the speed for the missile in km/h if it is to cover 5 km in 300 minutes.
- Calculate the possible angles of projecting the missile in the range $0^\circ \leq \theta \leq 360^\circ$ if it is to move at the speed obtained in (b) above.

Item 3

A large-scale grain milling and processing factory near the Mbale industrial zone is conducting a comprehensive operational audit to maximize its financial efficiency and reduce overhead expenses. The plant manager needs to evaluate three critical sectors of the business: raw grain procurement optimization, automated hydration control systems, and wholesale product inventory market readiness.

The factory's recent supply ledger shows three separate bulk batch purchases of Maize Flour, Millet Flour, and Sorghum Grain (measured in units of 100 kg bags) as detailed below:

Purchase Batch	Maize Flour (bags)	Millet Flour (bags)	Sorghum Grain (bags)
Batch A	2	1	2
Batch B	1	3	1
Batch C	3	1	1

The total financial expenditure for batches A, B, and C was 750,000/=,

1,000,000/=, and 800,000/= respectively. The manager will classify future raw material acquisitions as cost-friendly if a prospective order of 5 bags of Maize Flour, 4 bags of Millet Flour, and 2 bags of Sorghum Grain stays below a total budget of 2,100,000/=.

To maintain precise product moisture levels, the factory uses an automated processing bay where a liquid conditioning compound is introduced. Sensor telemetry records indicate that the volume of conditioning fluid consumed during the daytime production shift is represented by $\sqrt{(11x+75)}$ thousand litres, while the volume consumed during the night-time shift is $\sqrt{(x + 25)}$

thousand litres. The automated logs show that the total volume of fluid consumed over a full 24-hour cycle is exactly 20 thousand litres. The automated hydration system successfully protects the grains from spoilage without risking structural degradation only if the chemical variable x does not exceed 15 units.

Furthermore, the facility currently holds an inventory of 25 tons of processed flour sitting in its main storage reservoir. Due to regional market price volatility, the facility's inventory software implements a strict regulatory rule to automatically determine whether a stock batch is cleared for immediate external distribution or must be held back.

A batch is cleared for export only if 90 times the number of tons in storage is strictly greater than the square of the tonnage plus 1400. The plant manager needs to verify if the current stock level satisfies this safety margin to be cleared for the market.

Task:

Basing on mathematical evidence with justifications, advice the manager accordingly.

Item 4

A commercial fruit processing factory near the Natete industrial zone specializes in manufacturing concentrated juices. The factory relies on automated extraction chambers that operate under controlled conditions. The technical team observed that as the internal chamber pressure factor (x) increases, the efficiency percentage (E) of the extraction juice output decreases according to the polynomial model: $E = ax^2 + bx + c + 40$

Where a , b , and c are constant operation parameters determined by the fruit density.

The quality control log shows the following efficiency levels recorded at different pressure factors using the same batch configuration:

Pressure Factor (x)	Extraction Efficiency (E)
1	37
2	32
3	19

An external production consultant recommends that the factory should aim to run the extraction process at a pressure factor of $x=4$ to conserve power, claiming it will still maintain a viable efficiency.

To streamline the workflow before the pasteurization phase, the factory employs two heavy-duty extraction pumps, Pump P and Pump Q. The operational workloads of these pumps depend on the running time, t minutes, according to the mathematical models:

Pump P: 9^{t+1}

Pump Q: $28(3^t) - 9$

To prevent fluid congestion in the pipeline, both pumps must achieve a perfect equilibrium of workloads, and the operational time t required to reach this balance should not exceed 3 minutes.

Additionally, the factory stores its finished product in a multi-level automated warehouse consisting of 9 distinct storage zones. The total capacity of these zones is represented by the terms of the binomial expansion: $N = (x + 3)^8$ hundred crates.

Where x represents the baseline clearance height (in meters) allocated for stackable shipping crates. The factory management plans to expand their current distribution orders. This expansion will be approved only if the storage capacity calculated using the first four terms of the expansion with crates set at a clearance height of $x = 0.2$ m

shows that the facility can accommodate at least 550,000 crates.

Task:

- Help the factory officials to get the system of equations from the efficiency log to determine the constants a , b , and c , and analyse whether the consultant's recommended operating pressure yields a viable efficiency.
- Determine the operational time required for the two pumps to achieve equilibrium, and evaluate whether the warehouse capacity meets the criteria needed to approve the factory's distribution expansion plans.

Item 5

A certain boy dropped out of school due to lack of school fees. He decided to apply the skill of making soap that he acquired while still studying.

He contracted a welder to manufacture containers with square cross-section in which he is to mold bars of soap. The welder made a percentage error of 1% when measuring the side of the cross-section. The length of the bars of soap he wishes to mold is 10 times the side of the cross-section.

Under acceptable standards, only bars of soap whose percentage error in volume does not exceed 4% are allowed on the market. He also has square sheets of hard paper of dimensions 50cm by 50cm from which he wishes to make the biggest boxes for packaging the bars of soap. The box is to be made by cutting off equal squares from each corner of the rectangular sheet and then folding it so as to contain 10 bars of soap each of approximately 900cm³. He wishes to know whether the sheets will serve the purpose. He also conducted a market survey of the available bars of soap and discovered that they sold at a rate of:

$$dN/dt = (4t^2 + 13t + 7) / ((t + 1)(t + 2))$$

where N is hundreds of bars of soap sold after t days. From this survey, he will consider the business profitable if after every 7 days, he can sell more than 2000 bars of soap.

Task:

Using your skills in calculus, help the young entrepreneur to make valid decisions.

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