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EXTENDED RAIM (ERAIM): ESTIMATION of SV OFFSET

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BIOGRAPHY

James L. Farrell (Ph.D., U. of MD, 1967) is a member of IEEE and ION, a former AIAA local board member, a registered professional engineer in Maryland, and a member of TRIANGLE plus various scholastic honorary fraternities. Technical experience includes teaching at Marquette and UCLA, two years each at Minneapolis Honeywell and Bendix-Pacific, and 30 years at Westinghouse in design, simulation, and validation-test for modern estimation algorithms in navigation and tracking applications [e.g., F16 AFTI, B1 phased array radar, SDI; tank fire control system design for U.S. Army (ARDEC), generation of test data for validation of EW systems and for bench validation of B1 SAR mode; INS updating and transfer alignment algorithm design, development of programs for USAF-WPAFB (director fire control system evaluation) and for NASA (orbit & attitude determination and coupled rotational/structural deformation of Radio Astronomy Explorer); missile guidance optimization and MLE boundaries] plus digital communication system design (synchronization, carrier tracking, decode). He is author of the book, *INTEGRATED AIRCRAFT NAVIGATION* (Academic Press, 1976; now in its fifth printing) and of various columns plus over 50 journal and conference manuscripts. Active in RTCA (Washington D.C.) for several years, he is currently a co-chairman of the Fault Detection and Isolation Working Group (WG-5; FDI) within SC-159.

ABSTRACT

A method has been devised for extending RAIM. Whereas conventional RAIM excludes the suspect SV from the solution, ERAIM retains it while furthermore including its bias as a fifth unknown to be estimated. In a 5-SV snapshot both RAIM and ERAIM form five solutions, with each SV taking its turn as the suspected bias source. Extension to six SV's for fault isolation produces a parity scalar. Additional elaborations address complications such as multiple biased SV's, bias histories not limited to ramps, and further augmentation for time base differences in an integrated system.

INTRODUCTION

Under fairly general conditions, a single SV bias is observable, so that ERAIM solutions match those from conventional RAIM.[1] For the 5-SV case, the same set of five positions will be computed either way because, in each solution, every pseudorange but that from the offset SV is insensitive to the bias state. This produces a 5×5 matrix with one column having four zeros and a one. Inversion yields a 4×4 partition identical to the unaugmented solution.

Instead of "resting on the sidelines" the fifth SV serves as a bias indicator for ERAIM. Estimated SV offset is an adjusted value of the fifth residual; amount of adjustment is the perceived contribution caused by usage of *a priori* position and user clock values to form that residual. Four of the five candidate solutions will be biased due to modeling error and, although a formulation is provided for this modeling effect, it will not be apparent during operation which is the unbiased solution. That decision of course is assigned to the *isolation* portion of FDI (Fault Detection and Isolation). Again this will be done with the addition of a sixth satellite.

Fortunately this is one operation wherein an expanded scope facilitates the solution somewhat. **QR** decomposition (previously applied to RAIM by M. Brenner[2] and F. van Graas[3]) is directly applicable to ERAIM. With six SV's available there are six candidate solutions. Each one is overdetermined, providing estimates for five unknowns - the usual four + estimated bias for the suspect SV. Decomposition immediately produces a parity variable that can be used for identification. With one corrupted SV, every solution except one (the one with the offset SV used in the suspect role) will produce a parity variable having nonzero mean. All of the techniques and procedures relevant to statistical decisions based on parity, plus recent extensions[4] for confidence levels in the presence of parameter uncertainties, can be adopted here. This entire analysis is applicable to time-varying offsets, possibly representing combined error effects (*e.g.*, ephemeris as well as SV clock).

ANALYSIS

The development is subdivided into sections. In the first of these, notation and conventions are given for the usual (4 - SV) solution. Expansion to five SVs follows, wherein it is established that familiar expressions are preserved in their familiar form. This is followed by interpretations and operational considerations applicable to FDI algorithm development.

4 - SV CONFIGURATION

The notation $\boldsymbol{a}_i$ will denote the direction cosine vector corresponding to the i^{th} SV so that, in the usual 4 - dimensional solution,

$$\hat{x} = \mathrm{h}^{-1} z \tag{1}$$

$\mathbf{h}$ has the form,

$$\mathrm{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} a_1^T & \vdots & 1 \\ a_2^T & \vdots & 1 \\ a_3^T & \vdots & 1 \\ a_4^T & \vdots & 1 \end{bmatrix} \tag{2}$$

where z represents the residual vector expressed as

$$\boldsymbol{z} = \mathbf{h}\,\boldsymbol{x} + \epsilon \tag{3}$$

while, in accordance with standard Extended Kalman Filter (EKF) notation,

$$\boldsymbol{x} \triangleq \mathbf{X} - \hat{\mathbf{X}} \tag{4}$$

represents the "small difference of large numbers" departure between true and estimated state while ϵ denotes a zero-mean vector of random errors in pseudorange measurements.

5 - SV CONFIGURATION

The above familiar case will now be extended; for ease of illustration, the simplest extension will be considered first. Suppose that a fifth SV were known to have an offset (clock plus radial error) and it became necessary to estimate its value x_S. With five SV's available, Eq. (3) is replaceable by the 5 - dimensional vector relation,

$$\mathbf{z} = \mathbf{H}\,\mathbf{x} + \epsilon \tag{5}$$

in which

$$\mathbf{z} = \begin{bmatrix} \boldsymbol{z} \\ z_5 \end{bmatrix} \;;\quad \mathbf{x} = \begin{bmatrix} \boldsymbol{x} \\ x_S \end{bmatrix} \;;\quad \epsilon = \begin{bmatrix} \epsilon \\ \epsilon_5 \end{bmatrix} \tag{6}$$

and

$$\mathbf{H} = \begin{bmatrix} \mathbf{h} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \boldsymbol{h}_S & \vdots & 1 \end{bmatrix} \tag{7}$$

It is easily seen from direct multiplication that the inverse of $\mathbf{H}$ is

$$\mathbf{H}^{-1} = \begin{bmatrix} \mathbf{h}^{-1} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ -\boldsymbol{h}_S\,\mathbf{h}^{-1} & \vdots & 1 \end{bmatrix} \tag{8}$$

so that, when Eq. (1) is replaced by

$$\hat{\mathrm{x}} = \mathrm{H}^{-1}\,\mathrm{z} \tag{9}$$

substitution of Eqs. (6,8) produces the results,

$$\begin{bmatrix} \hat{\boldsymbol{x}} \\ \cdots \\ \hat{x}_S \end{bmatrix} = \begin{bmatrix} \mathbf{h}^{-1} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ -\boldsymbol{h}_S\,\mathbf{h}^{-1} & \vdots & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{z} \\ \cdots \\ z_5 \end{bmatrix} \tag{10}$$

The top partition and Eq. (1) are equivalent; thus augmentation to include SV offset leaves the original position-cum-user-clock solution *unchanged*. It easily follows that the original GDOP matrix (denoted here as $\mathbf{g} = (\mathbf{h}^T\mathbf{h})^{-1} = \mathbf{h}^{-1}\,\mathbf{h}^{-T}$ and all its implications (PDOP, HDOP, VDOP, TDOP) are likewise unaffected as are any other characteristics of the usual 4-SV formulation. Retention of familiar behavior is directly traceable to the decoupling effect of the null partition in the upper right of Eq. (8), which relieves position and user clock corrections from dependence on the estimated SV offset.

The solution for SV offset can be expressed in transparently obvious form by combining the lower partition of Eq. (10) with Eqs. (1) and (2):

$$\hat{x}_S^{(+)} = z_5 - \boldsymbol{h}_S\,\hat{\boldsymbol{x}} = z_5 - \hat{x}_4^{(+)} - \boldsymbol{a}_S^T \begin{bmatrix} \hat{x}_1^{(+)} \\ \hat{x}_2^{(+)} \\ \hat{x}_3^{(+)} \end{bmatrix} \tag{11}$$

Stated verbally, this last expression characterizes estimated SV offset as an adjusted value of the fifth residual; amount of adjustment is the perceived contribution caused by usage of *a priori* position and user clock estimates to form $\mathbf{z}$.

Superscripts $^{(+)}$ are used in Eq. (11) to emphasize the identity of state variable corrections as *a posteriori* estimates; contrast between roles of *a priori* (before processing) and *a posteriori* (after processing) quantities will attract further attention at appropriate points as the analysis proceeds.

Clearly this section is far from finished; the 5-SV development is thus far based on the condition that a *specific* SV is known to have an offset, while all other SV's in the solution are known to be bias-free. In operation it is of course necessary to form multiple candidate solutions, a procedure that has become standardized by a host of earlier studies in integrity. Among those multiple (in the immediate case, five) solutions, well-known properties of linear estimation make it apparent what to expect:

- In the absence of any SV bias, estimates will cluster within a space commensurate with geometry and RMS pseudorange measurement error σ_m.
- With a timing offset significantly larger than σ_m in one or more SV's, solutions will disperse. When only one SV has the offset, one of the solutions will be unbiased — but it will not be known which solution is that one.

The behavior just noted was reported in Ref. 5. Due to the aforementioned decoupling effect that behavior remains applicable here, unaltered by the explicit appearance of our fifth error state. To explore this while using the convenient notation

$$\mathbf{h}^{-1} = \begin{bmatrix} \alpha_1 & \vdots & \alpha_2 & \vdots & \alpha_3 & \vdots & \alpha_4 \\ \cdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ 1 - a_1^T\alpha_1 & \vdots & 1 - a_2^T\alpha_2 & \vdots & 1 - a_3^T\alpha_3 & \vdots & 1 - a_4^T\alpha_4 \end{bmatrix} \quad (12)$$

an unmodeled bias b in the j^{th} SV contributes a position error $b\,\boldsymbol{\alpha}_j$ and a false correction $(1 - \boldsymbol{a}_j^{\mathrm{T}}\boldsymbol{\alpha}_j)\,b$ in user clock bias.[§] There will be four sets of these unmodeled error contributions for one SV offset (each set of course having its own matrix $\mathbf{h}^{-1}$ — and therefore its own different quartet of vectors $\boldsymbol{\alpha}_j$).

To exemplify the performance just described, let SV #2 be the unidentified bias source. Repeated application of Eq. (10) will then produce

- one unbiased solution (obtained when SV #2 is used for the bottom row),

and

- four biased solutions, obtained when the top partition is made up of residuals from the following SV combinations: 1&2&3&4, 1&2&3&5, 1&2&4&5, 2&3&4&5. Each of these solutions will have a nonzero mean error, equal to the product of the bias b in the 2[nd] SV multiplied by the second column of a matrix $\mathbf{h}^{-1}$ — again with the reminder that the vector denoted $\boldsymbol{\alpha}_2$ will differ for each of the four combinations as previously noted.

First and second moments can now be given for estimation errors, in the case of one offset SV. Multivariate density functions for the five candidate solutions are characterized by

- one zero-mean random vector with covariance matrix $\sigma_m^2\mathbf{G} = \sigma_m^2(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1} = \sigma_m^2\mathbf{H}^{-1}\mathbf{H}^{-\mathrm{T}}$; using Eq. (8) with the notation ($-S$) from Ref. 5 to indicate absence of the subscripted SV from the 4×4 solution,

$$\mathrm{G}_{-S} = \begin{bmatrix} \mathbf{g}_{-S} & \vdots & -\mathbf{g}_{-S}\,h_S^T \\ \cdots & \vdots & \cdots \\ -h_S\,\mathbf{g}_{-S} & \vdots & 1 + h_S\,\mathrm{g}_{-S}\,h_S^T \end{bmatrix}; \quad \mathrm{g}_{-S} \triangleq \mathrm{h}_{-S}^{-1}\mathrm{h}_{-S}^{-T} \quad (13)$$

and

- four random vectors with nonzero mean (due to unmodeled bias b in the j^{th} SV, incorrectly attributed to the k^{th} SV) having the form,

$$<\mathbf{x}_{-\mathbf{k}}^{(+)}> = -\begin{bmatrix} \alpha_j \\ \cdots \\ 1 - a_j^T\alpha_j \\ \cdots \\ (a_j - a_k)^T\alpha_j - 1 \end{bmatrix} b, \quad j \neq k \quad (14)$$

with covariance matrices $\sigma_m^2\mathbf{G}$, where

$$\mathbf{G}_{-\mathbf{k}} = \begin{bmatrix} \mathbf{g}_{-\mathbf{k}} & \vdots & -\mathbf{g}_{-\mathbf{k}}\,\boldsymbol{h}_k^T \\ \cdots & \vdots & \cdots \\ -\boldsymbol{h}_k\,\mathbf{g}_{-\mathbf{k}} & \vdots & 1 + \boldsymbol{h}_k\,\mathbf{g}_{-\mathbf{k}}\,\boldsymbol{h}_k^T \end{bmatrix}; \quad \mathbf{g}_{-\mathbf{k}} \triangleq \mathbf{h}_{-\mathbf{k}}^{-1}\mathbf{h}_{-\mathbf{k}}^{-T} \quad (15)$$

[§] For adherence to the Kalman sign convention see Eq. (14).

If the pseudorange measurement errors are gaussian, the expressions just given are sufficient to define completely the multivariate density functions. For non-gaussian errors more information is required; although beyond the scope here, this will be necessary to investigate in a subsequent effort.

Strategy for detecting an SV offset is based directly on knowledge of these multivariate density properties. For the 5-SV case the estimate for x_S itself can be used as a test statistic, with the product $\sigma_m \sqrt{G_{55}}$ used to set a threshold according to a chosen alarm rate; a corresponding protection radius is established (*e.g.,* Ref. 3). Since there are five solutions in this example, the procedure is followed for all five, each with its own individual value of G_{55} . An alarm is declared when any of the five estimated SV offset values trips its threshold. This is not as conservative as it might initially appear, since a bias will tend to betray its presence in multiple solutions. Still, the existence of multiple test trials – with inputs that are partially correlated – can complicate the detailed parameter settings; nevertheless a method has been realized for direct detection of SV offset.

The foregoing statement, regarding the tendency of a bias to betray its presence repeatedly, can be viewed as both a blessing and a curse. By correctly influencing the unbiased solution *and* *in*correctly influencing the four biased solutions via Eq. (14), this tendency facilitates detection – while also clouding the identity of which SV produced the biased pseudorange. For that task we resort, as usual, to the addition of more information.

6 - SV CONFIGURATION

With six pseudoranges the estimation equation becomes a 6×1 vector relation,

$$\mathbf{z} = \mathbf{H}\,\mathbf{x}^{(-)} + \epsilon \tag{16}$$

wherein $\mathbf{z}$ and ϵ are 6×1 vectors while $\mathbf{x}^{(-)}$ still conforms to Eq. (6), and $\mathbf{H}$ is the 6×5 matrix,

$$\mathbf{H} = \begin{bmatrix} \boldsymbol{h} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \boldsymbol{h}_S & \vdots & 1 \end{bmatrix} \tag{17}$$

and $\boldsymbol{h}$ is a 5-row version of $\mathbf{h}$. ERAIM procedure in this case produces six overdetermined solutions for the same five variables first introduced – in the role of *a priori* states, as now noted – in Eq. (6).

Once again the null vector in the upper right partition enables separation of SV offset in the 5-state solution†

$$\mathbf{H}^{\#} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}} = \begin{bmatrix} (\boldsymbol{h}^{\mathrm{T}}\boldsymbol{h})^{-1}\boldsymbol{h}^{\mathrm{T}} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ -\boldsymbol{h}_S\,(\boldsymbol{h}^{\mathrm{T}}\boldsymbol{h})^{-1}\boldsymbol{h}^{\mathrm{T}} & \vdots & 1 \end{bmatrix} \tag{18}$$

but the procedure recommended here for an operational algorithm will exploit the powerful **QR** decomposition [Refs. 2,3]. With each SV taking its turn in the suspect role, the six solutions then produce six parity *scalars*, all of which are biased except one – the one with the biased SV as the suspect. Immediately the approach for fault isolation can be outlined thus:

- Express the mean estimation error resulting from bias b in one SV (*e.g.,* with $\mathbf{1}_j$ defined as the j^{th} column of the 6×6 identity matrix),

$$< \mathbf{x}^{(+)} > = -\, b\,\mathbf{H}^{\#}\mathbf{1}_j \tag{19}$$

- Express the covariance matrix for each value of k as $\sigma_m^2 \mathbf{G}_{-k}$, with

$$\mathbf{G}_{-k} = (\mathbf{H}_{-k}^{\mathrm{T}}\mathbf{H}_{-k})^{-1} = \tag{20}$$

$$\begin{bmatrix} (\boldsymbol{h}_{-k}^{\mathrm{T}}\boldsymbol{h}_{-k})^{-1} & \vdots & (\boldsymbol{h}_{-k}^{\mathrm{T}}\boldsymbol{h}_{-k})^{-1}\,\boldsymbol{h}_k^T \\ \cdots & \vdots & \cdots \\ -\boldsymbol{h}_k\,(\boldsymbol{h}_{-k}^{\mathrm{T}}\boldsymbol{h}_{-k})^{-1} & \vdots & 1 + \boldsymbol{h}_k\,(\boldsymbol{h}_{-k}^{\mathrm{T}}\boldsymbol{h}_{-k})^{-1}\,\boldsymbol{h}_k^T \end{bmatrix}$$

- Obtain **Q** and **R** from decomposition of **H** as in Refs. [2,3].

- Apply conservative adjustments to σ_m , to conform to a prescribed confidence level.

- Use values from the foregoing steps to derive thresholds for each parity variable[3].

- Determine protection radii[3] corresponding to the parameters obtained; these are the error levels to be associated with the alarm rates and probabilities stated for operation.

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† The nonsingular ($|\boldsymbol{h}^{\mathrm{T}}\boldsymbol{h}| \neq 0$) solution is given here; a singular case would be of limited usefulness for FDI. So would a larger number of 4-state solutions; the matrix **H** compacts all available information for the operation at hand.

There is of course no claim that this description is complete; much work obviously remains. What is intended here is the introduction of an approach for FDI, with the requisite analytical justification. ERAIM exploits established techniques (modern estimation, **QR** decomposition, statistical confidence level settings), to provide a direct means of identifying bias sources while making a judgment of their nature as well as their size (*e.g.,* inconsistent results from repetitive snapshots might indicate causes other than SV clocks). The remainder of this section compares ERAIM to other FDI methods and identifies possible extensions of the basic approach.

PAST and FUTURE FDI

ERAIM differs from an earlier FDI approach [6] by using **QR** rather than χ^2 and by setting thresholds from first- and second order moments { means and covariances; Eqs. (19) and (20) } rather than Monte Carlo simulation results. That observation is intended only to clarify a basic difference of approach, not to imply superiority of one over the other – no comparative study has been done. A comparison *is* being performed, however, involving ERAIM *vs* the parity vector space used in Ref. 7. If future results indicate a preference for ERAIM, it could replace the vector space approach as the baseline adopted by WG-5 of RTCA SC-159. Regardless of that outcome, many of the tools and methods used in Ref. 7 will be needed for the complete FDI algorithm. These include the diagnostics available from time histories of repeated snapshot solutions and, especially, the usage of Markov Chains to define recovery / repair sequences.

Extension of ERAIM to multiple failures is quite straightforward. For two biased SVs, another row would be added at the bottom of Eq. (7) for fault detection only, or to Eq. (17) for FDI. Instead of trying each SV as suspect, the method would try every possible SV *pair*. Performance would of course suffer relative to the single biased SV case, but that would be true of any approach. Note that, in principle, extension to any number of SV failures is conceptually straightforward. For FDI, the algorithm always calls for one more SV than

$$4 + (\text{Number of biased SVs})$$

until integration with another system with its own independent time base, as in Ref. 3. That of course calls for still one more time state. In all cases, however, there is always one parity variable; a parity *space* is replaced by a parity *scalar* plus direct estimation of any biases considered.

CONCLUSIONS

An extended RAIM (ERAIM) approach has been defined, forming direct estimates for SV offset. In addition to ease of interpretation afforded by this direct estimate, the approach offers unaltered retention of solution sets obtained from conventional RAIM. Thus, nothing is lost from the considerable RAIM literature already published. As an added benefit, a unified algorithm allows all FDI decisions to use a *scalar* parity variable (whereas conventional RAIM requires a parity vector for fault isolation). This latter feature becomes progressively more important as multiple SV failures are considered.

Future combination of ERAIM with techniques described in Ref. [4] will allow operational decisions to be supported by full mathematical rigor. This will substantiate the parameter values being quoted (*e.g.,* alarm rates, probabilities of detection, isolation, *wrong* isolation, etc.) – including firmly established levels of confidence that those parameters will not fall short of integrity requirements.

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