IMU COAST: NOT a SILVER BULLET

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Presented at Institute of Navigation 55th Annual Meeting
Cambridge MA June 1999

BIOGRAPHICAL

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Edwin McConkey (MSEE, BSEE + Mathematics, Univ of Michigan, 1964) is a registered professional engineer in the state of Florida. He is manager of the Technical Support Division for Air Transportation Systems Operation at SAIC, with 13 years of experience managing FAA. DOT, and NASA programs plus 34 years of technical and management experience in R&D, system engineering, and management support for air transportation system programs. Areas of expertise include navigation, air traffic control systems, helicopters, and advanced vertical flight aircraft. He has worked on R&D projects including GPS, MLS, and Loran-C, with emphasis on usage in the National Air Space (NAS).

ABSTRACT

Gyro misalignment, often overlooked, can produce potentially significant error accumulation in flight paths containing multiple turns. It is recommended that the analysis and MATLAB program presented herein be put to trial in tests with GPS data withheld during holding patterns – and that the results be considered in defining pertinent specifications.

INTRODUCTION

The Federal Aviation Administration has plans to decommission existing terrestrial navigation systems [Reference 1] after a nominal period (*e.g.*, five years) following full implementation of the Wide Area Augmentation System and Local Area Augmentation System. However, some concern remains about exclusive reliance on Global Positioning System (GPS)-based services and navigational backup options continue to be explored. Inertial Measurement Units (IMU's) in particular, have properties making it an intriguing backup option; no interference-prone external signals, prospects for low-cost IMU's in a few years, and many high-end users are already equipped. While intriguing, the use of IMU after a radionavigation outage should be cautiously implemented – especially for demanding applications in terminal airspace and for instrument approaches. This paper highlights an often overlooked IMU error (gyro misalignment), and proposes testing to characterize error bounds in typical outage scenarios.

PATH-RELATED LIMITS in COAST DURATION

A prospect often raised as a backup during GPS outage is inertial navigation – and it clearly is true that, when signals in space (SIS) are absent, coast is permissible for some duration. The navigation community is of course well aware of this capability. In fact, quite often this potential is extended – pre-dropout SIS information is used to improve estimates of inertial system dynamics, thus enabling better extrapolation. While the technique deserves its enthusiastic endorsement, it is prudent to raise some cautions. By labeling the method as a "calibration" the navigation community has bestowed on this approach credibility for a higher-than-warranted degree of versatility. As a result the method has often been stretched, conceptually, in applications involving extended outage durations without sufficient attention to IMU quality and flight scenarios.

A situation that would be particularly troublesome is SIS dropout followed by significant heading changes and subsequent coast (either straight or in a holding pattern), for several minutes. What makes this scenario difficult is its destruction of pre-dropout estimates of inertial information planned for use in subsequent extrapolations. In reality that information can be significantly degraded by the turning action - imperfect construction within an inertial measuring unit (IMU) assembly allows strapdown roll and pitch gyros, for example, to experience small fractions of the yaw rate. Lab measurements can remove much of the imperfection before IMU installation into the aircraft, but residual components remain (or can emerge from aging or thermal effects after all lab measurements); all development herein refers exclusively to that unknown remaining amount. Since the nav community has not given much attention to concerns expressed here it is first necessary to establish the importance of gyro misalignment in this application:

Superficially it might seem that cyclic heading changes intrinsic to a holding pattern would preclude error buildup in any consistent direction. However, reality is more complex, and several factors need to be considered:

- Azimuth cycling does produce alternating transformations of vehicle-based errors into nav coordinates – a subtle benefit that arises in analyzing drift effects in turns.
- In a holding pattern, however, the benefit just described extends to effective drift *rates* from gyro cross-axis effects; the corresponding misorientation angles never change sign, and velocity errors continue to grow.
- The amount of temporary effective drift rate from gyro misalignment can far exceed levels deemed tolerable for a high-performance IMU.

To substantiate the last remark, realize that IMUs with 0.01- deg / hr gyros, customarily regarded as nav-quality, could have gyro misalignment errors on the order of $100\ \mu$rad. During a 180^o / minute turn, that produces an effective drift rate more than a hundred times the value just mentioned; thus, *when there are repeated turns*,

- the benefit of the 0.01- deg / hr performance is thwarted, and
- it is inappropriate to characterize short-term drift rate propagation based on the cubic term from Eq. (5-88) on page 178 of Ref. 2 –

$$\textit{drift effect during } \textbf{cruise} = (\, g\, t^3 / 6\,) \bullet (\,\textit{steady drift rate}\,)$$

Rather than the cruise relation, an expression or an algorithm must be used that takes the flight path into account. That will be done here by an algorithmic solution for multiple cycles in a standard racetrack holding pattern.

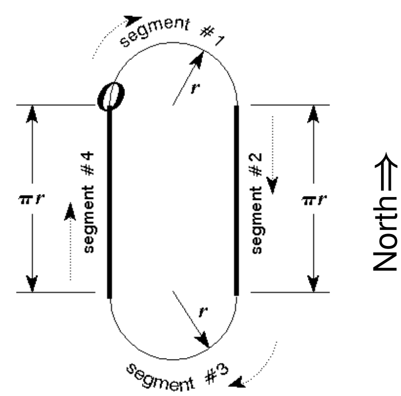

Fig. 1 : **Holding Pattern**

ERROR GROWTH in HOLDING PATTERNS

A holding pattern, as shown in Fig. 1, consists of 180° arcs separated by straight-and-level flight legs. Standard turn rates are 3°/ sec; thus it takes a minute to turn 180°. Because the straight flight legs have the same duration (t_L = 1 minute), their lengths are equal to the (half-circumference) excursion over the arc.

The development to follow, supported by the analysis in APPENDIX A and by the MATLAB numerical integration program in APPENDIX B, is selectively focused on gyro sensitive-axis direction offsets during holding patterns.

Both roll and pitch gyros sense a fraction (f_1 and f_2 respectively) of the yaw rate. Instantaneous error in apparent *attitude* about level axes will then be essentially proportional to the time integral of that yaw rate – which corresponds to the total rotational excursion accumulated while that yaw rate is maintained – for small excursion angles. Reasons for the qualifying expression "essentially" and the small-angle caveat involve rotation of roll and pitch axes during the azimuth rotation. A directional change of θ radians, then, will produce leveling errors of essentially $f_1 \theta$ and $f_2 \theta$, if $\theta << 1$. For larger angles that characterization is conservative but not outlandish; APPENDIX A shows that $\theta = \pi$ produces attitude errors of $2f_1$ and $2f_2$ with respect to stable reference axes. Immediately this gives prompt insight into the error-generating process:

An aircraft initially heading North from point ***O*** in Fig. 1, with zero initial error in position, velocity, and angular orientation, undergoes a 180^o directional change as segment #1 of a holding pattern. With nominal values $f_1 = f_2 = 0.001$ adopted for simplicity of this introductory example, it follows from the preceding discussion that both roll and pitch attitude will be in error nominally by 2 milliradians (mr) at the end of that turn. Although the growth from 0 to 2 mr follows a curved time-history (corresponding to trigonometric projection of gyro sensitive axes on geographic reference directions), a linear ramp can approximate that growth in attitude error during the turn – yielding average leveling errors of 1 mr during segment #1. For (Southbound) segment #2 of Fig. 1, there is of course no rotation – therefore no increase nor decrease in tilts – which thus hold constant at the 2-mr values reached at the end of segment #1. During segment #3 the gyro axes – and therefore effective drift rates – reverse direction; the leveling errors thus revert to zero, staying constant at zero throughout (Northbound) segment #4. Averaging over the four segments, mean leveling errors for a full holding pattern cycle are then

$$(1 + 2 + 1 + 0) / 4 = 1 \quad \text{mr}, \qquad f_1 = f_2 = 0.001$$

and, more generally, it is now clear that averaged tilt is equal to corresponding misalignment f – which for a nominal 1-g lift, is essentially synonymous with an acceleration error of fg. In the short term (*e.g.*, for a duration t up to ten minutes) this clearly produces a position error $\frac{1}{2} fg t^2$.

For longer durations, substitute Earth Radius R and Schuler rate $W = \sqrt{g / R}$ rad / sec into Equation (3-51) of Ref. [2] :

$$\text{position error } = fR \, (1 - \cos Wt)$$

which is easily seen to approach

- $\frac{1}{2} fg t^2$ when $Wt << 1$.
- $f R$ when $\cos W t \Rightarrow 0$ (near 21 minutes) – nominally after five holding pattern cycles.

To assess the impact, three IMU classes will be addressed, from the standpoint of these uncorrected cross-axis projections:

- <u>Very High-precision</u> 10 parts per million (ppm); approx. 2 arc-sec; $f = 0.00001$.
- <u>Mid-range</u> 100 ppm; approx. 20 arc-sec; $f = 0.0001$.
- <u>Low-cost</u> 1000 ppm; 1 milliradian; $f = 0.001$ – or <u>more</u>.

Immediately it is clear that, after five holding pattern cycles, position error is of order 60 meters, 600 meters, or 6 Km for a high-precision, mid-range, or low-cost IMU, respectively. First of all, obviously, ***a low-cost IMU is not meant for coasting***. While that should not come as any surprise, it is appropriate to document an unequivocal statement to that effect. A more worrisome issue involves the intermediate-quality IMU's 600-meter performance – which violates NPA containment requirements. A crucial question concerning IMUs ***now being used in the NAS*** can be stated as follows: Can it be assured, with risk levels low enough for containment, that gyro cross-axis errors of these operational IMUs will not reach 100 ppm? The 2 arc-sec cross-axis error is acceptable, but what are the statistics?

This last point draws attention to requirements. Some strapdown IMU specifications fail to address gyro cross-axis sensitivity, and many others are vague about the amount – especially regarding relation to statistical population models (1-σ, 3-σ, $\cdots$). Obviously a "typical" value cannot be relied upon to deliver performance in critical situations. It is hoped that the case has been made here for clear specifications.

The prospect is also raised for prolonged durations of straight-and-level coast, beginning after a procedure that produces 180^o net change in direction. This case is modeled by the development already presented – but with the first turn (segment #1 of Fig. 1, producing 180^o net directional change) followed by an extended segment #2, M minutes in length. For durations up to ten minutes the dominant error, accumulated in segment #2, is essentially

$$\tfrac{1}{2} (2fg) (60M)^2 = 3600 fg M^2$$

or about $0.35 \bullet 10^5 M^2 f$ meters.

For high-precision, mid-range, and low-cost strapdown IMUs the dominant error just described is essentially $0.35 M^2$, $3.5 M^2$, and $35 M^2$ meters, respectively. With longer durations, the expression $2 f R\ (1 - \cos \Omega t)$ is used. Once again the expressions are simple enough to justify immediate interpretation. It is readily seen, for example, that

- Low-cost IMUs can yield over 1 Km six minutes after the turn.
- Mid-range error exceeds 1 Km well in advance of 21 minutes (as $\cos \Omega t \Rightarrow 0$).

Although the prospect of near straight-and-level coast after a turn is somewhat atypical, it is by no means extreme; it remains relevant since systems must perform despite unusual circumstances. In any event it only fortifies the point already made with racetrack cycles only: ***dependable IMU coasting during holding patterns will impose demands for definite specifications regarding cross-axis gyro errors.***

Results of the previously mentioned MATLAB program (APPENDIX B), supported by the racetrack path analytical development (APPENDIX A), are presented next. Accompanying plots show results for representative parameter values; subsequent discussion provides interpretation

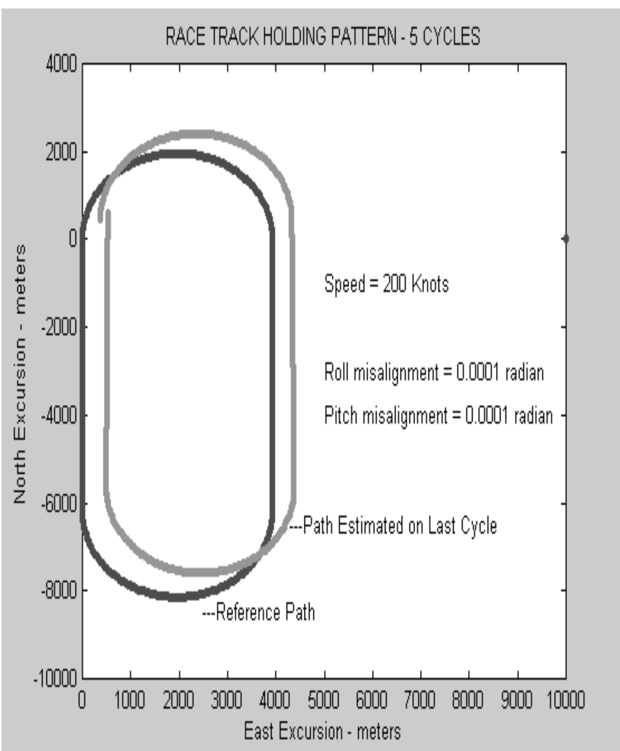

Fig. 2: **5 - CYCLE (20 - Minute) COAST – Last Cycle**

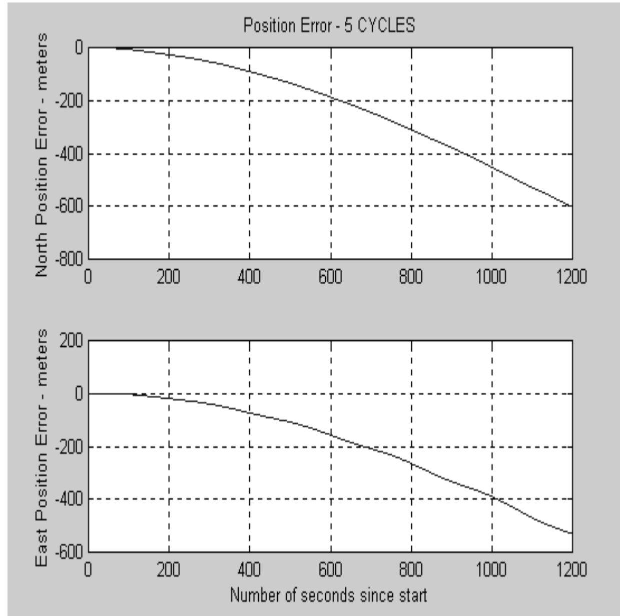

Fig. 3: **5 - CYCLE (20 - Minute) coast – Time-history**

Error reduction due to the Schuler effect is present in Fig. 3, in agreement with the simple analytical expression already shown for position error; thus each final component of horizontal error after five racetrack cycles is close to 600 meters (roughly 0.0001 × the Earth radius, at essentially a quarter Schuler cycle, rather than the 720-meter value that the quadratic expression would have produced). After eight racetrack cycles the error components exceed a kilometer.

It is reiterated here that *all* errors – position, velocity, and attitude – were initialized at zero; these results follow a perfect initial "calibration." To check validity of the behavior described here a test plan can obviously be introduced whenever desirable.

INTERPRETATION

Results just shown were obtained by numerically stepping through sixty 1-second intervals for each flight leg in a 5-cycle (= 20-minute) racetrack scenario (since rerun with time step doubled produce consistent results, the 1-second intervals are short enough for accuracy). Both initial values and initial slopes on the time-histories exhibit the appropriate zero initialization. The plots show major trends but not all details of every time-history can be seen clearly. Peak misorientation, for example, occurs 45 degrees before each half-cycle of heading, which explains that waveform's "overshoots" just before even segments begin. There are cyclic changes in higher derivatives of North and East errors (due, of course, to heading reversals) – but velocity errors are cumulative in nav-reference axes without changing sign.

Supplementary run plots showed additional cyclic constituents of error, with North and East components out of phase by a quarter-cycle — but the trends (in agreement with the expression given at the outset) are far more important as performance criteria. It was also verified, not surprisingly, that the bank angle had little effect on numerical results — which would interest those wanting to use closed-form analyses. The same misalignment error was assumed for both roll and pitch; when runs were made with only one, position errors in one axis was unchanged while the other was smaller by orders of magnitude. Rather than *RSS* the errors in the two directions, it is appropriate to note that each channel has essentially the same time history — thus the value in one axis can legitimately represent nominal cross-track position error magnitude at any time.

Quantitative values for error levels can be scaled, since they are proportional to misalignment and – up to almost twenty minutes – to the square of elapsed duration t (thus racetrack cycle count). It is seen that $\frac{1}{2} f g t^2$ is a reasonable characterization for parameter values used here. The 100-μrad misalignment, essentially 20 arc-seconds, might seem too pessimistic to represent an operational state-of-the-art (not low-cost) IMU — until it is recalled that reliance on coast performance calls for conservatism; there is minimal margin for unfulfilled promises when coast is really needed. If better misalignment can be guaranteed, let IMU specifications state it as a commitment. Scaling can then predict correspondingly better accuracy during the holding pattern.

It might be argued that, even with thermal and aging effects, gyro misalignment error can be *substantially* better than the 20 arc-second level cited here – thus the figures just shown are *too* pessimistic. Even if that case can be made, however, it would be (1) unconvincing without support by specifications, and (2) subject to further consideration in the event of a straight flight segment, several minutes in duration, after the holding pattern.

This is where an initially aligned IMU can still fall short of desired performance — a worst-case distance on the order of 40 nautical miles, for straight (or nearly straight) coast after the holding pattern, translating into a 12-minute duration following the hold. That puts performance into the realm discussed before the plots – wherein the effective error growth coefficient is doubled $\{f g t^2 = 9.8 f \cdot (12 \cdot 60)^2 \doteq 5 \cdot 10^6 f$ meters$\}$ – aggravated further by nonzero initial errors incurred during the holding pattern.

One additional observation is appropriate before this interpretive discussion is closed. The analysis and computer program used to substantiate the conclusions offered here are further supported by a much earlier, and far more extensive, effort. [3] A strapdown gyro triad was fed with myriad error sources, separately and in combination, with the output applied to repetitive ($200{,}000$ in these cases) attitude computer iterations. Of immediate interest here is Table 5 on page 1346 of Ref. [3]. The first seven cases in that table involve essentially constant angular rate, producing a final error characterized by a dominant term equal to the product of that rate (in this case, 1 rad/sec) multiplied by the run duration (100 sec) × (misalignment; unity in this case, since error is expressed in milliradians) × $\cos$ (*angle between angular rate axis and drift rate axis*) – and, finally, amplified by $\sqrt{2}$ (in view of the error formulation at the end of the Appendix to that reference). A subtle point is the behavior shown for the first three cases, wherein that dominant error vanishes because the axes just described are perpendicular; nevertheless the observed error is not zero. By application of reasoning used in the holding pattern analysis herein, average error throughout these first three cases should be 1 mr, $\sqrt{2}$ mr, and $\sqrt{2}$ mr, respectively; amplification by $\sqrt{2}$ as previously explained produces the values of error E tabulated in Ref. [3] for those three cases.

CONCLUSIONS

If the Federal Aviation Administration or selected users should decide that IMU's can provide a backup in the case of rare outages, it is recommended that tests be conducted to characterize IMU error bounds under outage scenarios that could arise in operation. One challenging scenario would keep a user, during an outage, in a holding pattern for several turns before clearance for a descent. Confirmation of results described herein – particularly violation of NPA containment requirements after five racetrack cycles – should result in more definite specification limits for gyro cross-axis errors.

REFERENCES

1. Federal Radionavigation Plan, 1998.
2. Farrell, *Integrated Aircraft Navigation*, Academic Press, 1976.
3. Farrell, "Performance of Strapdown Inertial Attitude Reference Systems," *AIAA Journal of Spacecraft and Rockets*, Sept. 1966, pp. 1340-1347.

APPENDIX A: ANALYSIS

At any time during a turn, vehicle rate error consists of a roll drift rate $f_1 \pi / t_L$ and a pitch drift rate $f_2 \pi / t_L$ rad/sec; these rate errors of course exist *only* during turns, vanishing while in the straight segments. With each cycle (racetrack excursion) subdivided into four flight legs (curved•straight•curved•straight – turns occurring in the odd-numbered segments), attitude errors in those segments receive contributions $(\pi f_1 / t_L)\, dt$ and $(\pi f_2 / t_L)\, dt$ from roll and pitch, respectively, for each numerical integration interval dt. Accumulation of this is properly performed with usage of the instantaneous transformation $\mathbf{T}_{G/A}$ from aircraft axes to platform (here, geographic) coordinates – obtained as the transpose of the inverse transformation $\mathbf{T}_{A/G}$ resulting from a roll angle[§]

$$Roll = \text{Arctan}\{ speed \bullet (\pi / t_L) / g \}, \quad g = 9.8 \text{ meters/sec}^2 \qquad [\text{ turning flight legs only }]$$

in combination with heading (Hdg) in a constant cardinal direction (during straight segments) or linearly increasing with time (during turning segments);

$$\mathbf{T}_{A/G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(Roll) & \sin(Roll) \\ 0 & -\sin(Roll) & \cos(Roll) \end{bmatrix} \begin{bmatrix} \cos(Hdg) & \sin(Hdg) & 0 \\ -\sin(Hdg) & \cos(Hdg) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Specific force in a coordinated turn is essentially an upward (thus negative) component with magnitude $g \operatorname{secant}(Roll)$ along the yaw axis – which is transformed through $\mathbf{T}_{G/A}$,

$$\mathbf{A} = \mathbf{T}_{G/A} \begin{bmatrix} 0 \\ 0 \\ -g / \cos(Roll) \end{bmatrix}$$

Operationally the strapdown IMU measures the specific force along vehicle axes and reexpresses it using the *apparent* transformation from aircraft axes to geographic coordinates – which, from Eq. (3-17) of Ref. 2, can be represented by the expression

$$(\mathbf{I} - \boldsymbol{\psi} \times) \, \mathbf{T}_{G/A}$$

where $\boldsymbol{\psi}$ contains the cumulative drift effect, transformed into platform coordinates[†] plus a Schuler effect represented here by the first term on the right of Eq. (3-39) from Ref. [2]:

[§] The analysis presupposes a coordinated turn, and factors exerting only minor influence (such as angle of attack, sideslip, crosswind) are ignored.

[†] $\boldsymbol{\psi}$ represents a small-angle approximation for expressing 3-dimensional misorientation between true and estimated attitude. Ref. 2 *pp* 120-123 contains a thorough analysis including, in addition to drift effects, gradual rotation of the nav reference frame and also the imperfections in that gradual rotation. Drift effects are dominant in the immediate analysis.

$$\boldsymbol{\psi}_{new} = \boldsymbol{\psi}_{old} + \mathbf{T}_{G/A} \begin{bmatrix} Roll\ drift\ rate \\ Pitch\ drift\ rate \\ 0 \end{bmatrix} dt \quad + \quad \begin{bmatrix} -East\ velocity\ err \\ North\ velocity\ err \\ 0 \end{bmatrix} \quad \frac{dt}{Earth\ radius}$$

The vector cross product $\boldsymbol{\psi} \times \mathbf{A}$ is the dominant contributor here to the time derivative of velocity vector error; thus in the MATLAB program (APPENDIX B) it is accumulated for each numerical integration interval dt.

In the **INTERPRETATION** section it was noted, not surprisingly, that numerical results are largely unaffected by the shallow bank angle experienced in holding patterns. Ignoring the distinction between heading and yaw, therefore, the dominant contributor to the forcing function for $\boldsymbol{\psi}$ [last term in Eq. (4-73) of Ref. 2] can be closely approximated as

$$\mathbf{T}_{G/A} \begin{bmatrix} Roll\ drift\ rate \\ Pitch\ drift\ rate \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(Hdg) & -\sin(Hdg) & 0 \\ \sin(Hdg) & \cos(Hdg) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \dot{H} \\ f_2 \dot{H} \\ 0 \end{bmatrix} = \begin{bmatrix} f_1 \cos(Hdg) - f_2 \sin(Hdg) \\ f_1 \sin(Hdg) + f_2 \cos(Hdg) \\ 0 \end{bmatrix} \dot{H}$$

or, multiplying by dt and integrating from zero to π,

$$d\boldsymbol{\psi} = \begin{bmatrix} f_1 \cos(Hdg) - f_2 \sin(Hdg) \\ f_1 \sin(Hdg) + f_2 \cos(Hdg) \\ 0 \end{bmatrix} d(Hdg) \ ; \ \Delta\boldsymbol{\psi} = \left. \begin{bmatrix} f_1 \sin(Hdg) + f_2 \cos(Hdg) \\ -f_1 \cos(Hdg) + f_2 \sin(Hdg) \\ 0 \end{bmatrix} \right|_0^{\pi} = \begin{bmatrix} -2f_2 \\ 2f_1 \\ 0 \end{bmatrix}$$

which demonstrates the effect noted in the section where the holding pattern (Fig. 1) was introduced: a rotation of π radians causes a cross-axis error of $2f$ radian with respect to a stable reference (instead of πf radian – which a small-angle assumption would have produced). Secondarily this development also illustrates the presence of additional terms which, despite averaging to zero, create “bumps and wiggles” in the time histories for leveling errors – and therefore in the error dynamics for velocity and position, which depend on those leveling errors.

Verticality error in each of two level axes after a turning segment can thus be characterized accurately here as $2f$ radian where f will be given values 10^{-5}, 10^{-4}, and 10^{-3} as previously discussed.

APPENDIX B: MATLAB Program for Racetrack Holding Pattern

```
grav=9.8;                  % gravity in meters/sec/sec
N=input('Enter # of racetrack cycles   ');
M=4*N;    % # of segments (semicircle or straight flight legs)
tleg=60.;                  % duration of each segment
delt=1.;
fps=(6076./3600)*input('Enter speed in Kts  ');
speed=fps*12./39.37;       % meters/sec
Iq=round(tleg/delt);       % # of steps per segment
I=M*Iq;                    % total # of steps
f1=(1.e-5)*input('Enter ROLL  misalignment multiplier   ')
f2=(1.e-5)*input('Enter PITCH misalignment multiplier   ')
rerr = zeros(3,1);
verr = zeros(3,1);
Y    = zeros(3,1);
vx = zeros(Iq,1);
vy = zeros(Iq,1);
rx = zeros(Iq,1);
ry = zeros(Iq,1);
qd  = zeros(I,1);
hdg = zeros(I,1);
xe = zeros(I,1);
ye = zeros(I,1);
vex= zeros(I,1);
vey= zeros(I,1);
Yx = zeros(I,1);
Yy = zeros(I,1);
for I=1:I,
  k=1+rem(I-1,Iq);         % time within current segment
  iq=1+fix((I-1)/Iq);      % # of segments entered thus far
  qd(I)=1+rem(iq-1,4);     % current segment (1, 2, 3, or 4)
  j=1+rem(I-1,Iq*4);       % time within current cycle
  ir=1+fix((I-1)/(Iq*4)); % current cycle = # of cycles entered
  if qd(I)==1,
    cR=1./sqrt(1.+(speed*(pi/tleg)/grav)^2);
    sR=sqrt(1.-cR^2);
    hdg(I)=pi*k*delt/tleg;
    cH=cos(hdg(I));
    sH=sin(hdg(I));
    TAP=[1 0 0; 0 cR sR; 0 -sR cR ]*[cH sH 0; -sH cH 0; 0 0 1 ];
    Y = Y + TAP' *(pi*delt/tleg)* [ f1 ; f2 ; 0 ] ;     % "psi"
    if I<=Iq*4
      vx(j)=speed*cH;
      vy(j)=speed*sH;
      rx(j)=(tleg*speed/pi)*sH;
      ry(j)=(tleg*speed/pi)*(1.-cH);
    end
  elseif qd(I)==2,
    cR=1.;
    sR=0.;
    hdg(I)=pi;
    cH=-1;
    sH=0;
```

```
    TAP=[1 0 0; 0 cR sR; 0 -sR cR ]*[cH sH 0; -sH cH 0; 0 0 1 ];
    if I<=Iq*4
      vx(j)=-speed;
      vy(j)=0;
      rx(j)=-speed*k*delt;
      ry(j)=speed*2*tleg/pi;
    end
  elseif qd(I)==3,
    cR=1./sqrt(1.+(speed*(pi/tleg)/grav)^2);
    sR=sqrt(1.-cR^2);
    hdg(I)=pi*(k*delt/tleg - 1.);
    cH=cos(hdg(I));
    sH=sin(hdg(I));
    TAP=[1 0 0; 0 cR sR; 0 -sR cR ]*[cH sH 0; -sH cH 0; 0 0 1 ];
    Y = Y + TAP' *(pi*delt/tleg)* [ f1 ; f2 ; 0 ] ;     % "psi"
    if I<=Iq*4
      vx(j)=speed*cH;
      vy(j)=speed*sH;
      rx(j)=(tleg*speed)*(sH/pi-1.);
      ry(j)=(tleg*speed/pi)*(1.-cH);
    end
  elseif qd(I)==4,
    cR=1.;
    sR=0.;
    hdg(I)=0;      % cH=cos(hdg(I))=1    and   sH=sin(hdg(I))=0
    TAP = [ 1 0 0 ; 0 cR sR ; 0 -sR cR ] ;
    if I<=Iq*4
      vx(j)=speed;
      vy(j)=0.;
      rx(j)=speed*(k*delt-tleg);
      ry(j)=0;
    end
  end
  Y = Y + (delt / 6378137) * [ -verr(2); verr(1); 0 ] ;   % Schuler
  Yx(I) = Y(1);
  Yy(I) = Y(2);
  A = TAP' *[ 0 ; 0 ; -grav/cR ];   % g/cos(ROLL) along yaw axis
  YxA=[Y(2)*A(3)-Y(3)*A(2);Y(3)*A(1)-Y(1)*A(3);Y(1)*A(2)-Y(2)*A(1)];
  rerr = rerr + delt * verr / 2. ;
  verr = verr + delt * YxA ;
  vex(I) = verr(1);
  vey(I) = verr(2);
  rerr = rerr + delt * verr / 2. ;
  x(ir,j) = rx(j) - rerr(1) ;
  y(ir,j) = ry(j) - rerr(2) ;
  xe(I) = rerr(1) ;
  ye(I) = rerr(2) ;
end
```

```
subplot(2,2,1),plot(Yx)
subplot(2,2,2),plot(Yy)
subplot(2,2,3),plot(vex)
subplot(2,2,4),plot(vey),pause
% plot(vx,'*',vy,'o'),pause
% AXIS([0. 10000. -8000. 2000.]);
ry(Iq)=10000.; % fictitious point prevents unequal-axis distortion
if N==8, ry(Iq) = 12000.;, end
axis('square')
subplot(111),plot(ry,rx,'.r',y(ir,:),x(ir,:),'.g')
grid off
title(['RACE TRACK HOLDING PATTERN - ',int2str(N),' CYCLES'])
xlabel('East Excursion - meters')
ylabel('North Excursion - meters')
text(2500,-8500,'---Reference Path')
text(5000,-1000,['Speed = ',num2str(fps*3600./6076.),' Knots'])
text(5000,-3000,['Roll misalignment = ',num2str(f1),' radian'])
text(5000,-4000,['Pitch misalignment = ',num2str(f2),' radian'])
text(4300,-6500,'---Path Estimated on Last Cycle'),pause
[X,map] = capture(1);
imwrite(X,map,'racetk.pcx')
axis('normal')
subplot(211),plot(xe)
grid on
title(['Position Error - ',int2str(N),' CYCLES'])
ylabel('North Position Error - meters')
subplot(212),plot(ye)
grid on
xlabel('Number of seconds since start')
ylabel('East Position Error - meters')
[X,map] = capture(1);
imwrite(X,map,'poserr.pcx')
```